William Peſt. 1777.

# A
# TREATISE
## OF THE
# THEORY and PRACTICE
## OF
# PERSPECTIVE.

WHEREIN
The Principles of that moſt uſeful Art, as laid down by Dr. BROOK
TAYLOR, are fully and clearly explained, by Means of moveable
Schemes, properly adapted for that Purpoſe.

The whole being deſigned as
An Eaſy Introduction to the Art of DRAWING in PERSPECTIVE,
AND
Illuſtrated by a great Variety of Curious and Inſtructive EXAMPLES,
Engraved by the AUTHOR.

---

> A Figure in a Picture, which is not drawn according to the Rules of Perſpective, does not repreſent what is intended, but ſomething elſe: ſo that it ſeems to me, that a Picture which is faulty in this Particular, is as blameable, or more ſo, than any Compoſition in Writing which is faulty in point of Orthography or Grammar. Dr. Brook Taylor's Preface.

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By DANIEL FOURNIER,
Drawing-Maſter, and Teacher of Perſpective.

The SECOND EDITION, Corrected, and greatly Enlarged.

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LONDON,
Printed for the AUTHOR, and ſold at his Houſe, in Wilde-court, near Great Queen-
ſtreet, Lincoln's Inn Fields:
Alſo by Mr. NOURSE, Bookſeller to His Majeſty, in the Strand.
MDCCLXIV.

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### THE

# PREFACE.

*AS Dr. Brook Taylor's Perſpective is undoubtedly the moſt excellent performance of its kind hitherto made public, I choſe rather to attempt an explanation of his principles, than to offer any thing of my own, with regard to the theory of this moſt uſeful art; and chiefly becauſe the generality of thoſe who apply themſelves to the ſtudy of that treatiſe, without being firſt acquainted with geometry, find great difficulty in conceiving the author's deſign, as all his ſchemes are drawn in plano: I have, to obviate this difficulty, introduced moveable ſchemes in the following work, ſo contrived as to be raiſed up and placed according as the nature of the problem may require. By theſe means the mathematical reader will, without farther trouble, eaſily conceive the manner of reſolving the ſeveral problems in the theory: but for the accommodation of thoſe who have not been accuſtomed to the ſcience of Perſpective in this manner, I have added the repreſentation in Perſpective of great part of the apparatus I made uſe of, when I gave lectures on that ſubject. For this I have found by experience in teaching, that the young practitioner, not uſed to mathematical demonſtration, more eaſily conceives this manner of explanation.*

*In the practical part, which begins at page 38, I have defined and made uſe of the terms, Ground line, Horizontal line, Point of Sight, &c. This I did for the conveniency of thoſe who may have already learned the theory of perſpective from ſuch authors as have treated thereon agreeable to theſe definitions; and becauſe artiſts commonly uſe the horizontal line as the boundary of the picture, for the ſame reaſon I have in the practice changed ſome of the terms from what they are in the theory; as for inſtance, Center is ſometimes called Point of Sight; and what Dr. Taylor calls Vaniſhing Point, Ozanam and others call Accidental Point; and notwithſtanding this difference in the definitions, the principles upon which their reſpective ſyſtems of Perſpective are founded, are exactly the ſame.*

*It muſt, indeed, be confeſſed that Dr. Taylor's method of treating this ſcience is preferable to any other, yet in order to render my treatiſe of general uſe, I thought it neceſſary to the practice to make the alterations above mentioned. I have alſo ſelected from Ozanam the moſt uſeful ſubject relating to ſhadows; and have, in my examples, much enlarged his ſchemes, and properly adapted the demonſtration of that author to Dr. Taylor's method; nor have I ſpared any pains to render the doctrine of ſhadows, which has hitherto been looked upon as the moſt difficult, as well as the moſt curious part of Perſpective, as plain as the nature of the ſubject and my abilities would permit.*

*"As the art of perſpective conſiſts in the knowledge of truly repre-"ſenting an object according to its natural appearance, ſo whatever is not "drawn agreeable to the rules thereof, cannot poſſibly repreſent what is "intended, but ſomething elſe: it is true, that as the art of painting "conſiſts in two parts, the inventive and the executive; the former may "indeed be varied according to the taſte and genius of the artiſt, in the "diſpoſition of the parts of his ſubject; but the latter is wholly confined "and ſtrictly tied to the rules of art, and therefore in this the artiſt is "not to take any liberties whatſoever; for what is perfectly juſt in the "real original objects themſelves, can never appear defective in a picture "where the objects are repreſented according to the true rule of Per-"ſpective". See Dr. Taylor's Preface.*

*Notwithſtanding the principles upon which Dr. Taylor has founded his treatiſe ſeem ſo very obvious as not to admit of any objection; yet ſome modern writers upon this ſubject have ſo far deviated from his theory, as to inform us, that a circle, when viewed in an oblique poſition, cannot poſſibly take the appearance of a true mathematical elipſis. I ſhall not at preſent enter into a confutation of the reaſons they give to ſupport the truth of this extraordinary aſſertion, but only point out wherein (I think) they are miſtaken: that the oblique ſection of a cone may be an elipſis, is certainly true. I ſay may be an elipſis, becauſe in ſome caſes it may be an hyperbola; and indeed in one particular caſe, viz. the ſubcontrary poſition in the oblique cone, the figure of the ſection will be a circle, (as we* *have ſufficiently proved at page 32.) Wherefore, if the cutting plane be ſuppoſed to be the plane of the picture, the repreſentation of a circle thereon muſt be either a circle, elipſis, or a right line, and that according to the different ſituation of the original circle with regard to the plane of the picture; if, after having found the eliptical repreſentation of an original circle we find that of its center, it will not coincide with the center of the elipſis; and this I apprehend may have deceived thoſe who have imagined the oval repreſentation of an oblique circle would not be of equal curvature in oppoſite parts of its circumference.*

TABLE,

# TABLE,

Shewing where to find the Explanation of every Plate.

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OF

# O F

# LINEAR PERSPECTIVE.

IN order that the learner may form a clear idea of the inveſtigations relating to the theory of *Perſpective* contained in the enſuing pages, it may be neceſſary to premiſe the following geometrical Definitions and Problems.

## DEFINITIONS.

1. *Magnitude* is whatever is extended, and therefore may be conceived to be contained or limited by ſome certain term or terms.

2. A *point* is that which has no parts, conſequently of itſelf no magnitude, and may be conſidered as indiviſible.

3. A *right line* is the neareſt diſtance between two points.

4. If two right lines inclined to each other be produced till they meet, they will form a plane or *right lined angle*. Thus the *angle* formed by the lines BD, CE, Plate I. Fig. 1. produced, is denoted by BAC.

N. B. An angle being generally denoted by three letters, the middle one always repreſents the angular point.

5. When a ſtrait line ſtands on another, ſo that it does not incline to one end more than the other, but makes the angles on both ſides equal; then are theſe called *right angles:* and the lines are ſaid to be *perpendicular* to each other; as BD, Plate I. Fig 2. is perpendicular to FE.

6. Every angle greater than a right one, is called an *obtuſe angle*; and every angle leſs than a right angle, is called an *acute angle*.

7. A plain ſurface terminated by three right lines, is called a *plain triangle.*

8. When the three ſides of a triangle are equal, it is called an *equilateral triangle.*

9. When only two ſides are equal, an *iſoſceles triangle.*

10. When the three ſides are unequal, a *ſcalene triangle.*

11. When one angle of a triangle is a right angle, it is called a *right angled triangle.*

12. Any one ſide of a triangle may be called the *baſe*, and the angular point oppoſite to the baſe is called the *vertex.*

13. In a right-angled triangle, the ſide oppoſite to the right angle is called the *hypothenuſe.*

14. A plane contained by four right lines, is called in general a *quadrilateral* figure; and when the four ſides are equal as well as the four angles, it is called a *ſquare.*

15. If the four angles are equal, and only the oppoſite ſides, a *rectangle.*

16. If the oppoſite angles are only equal, and the oppoſite ſides, a *parallelogram.*

17. A plain figure of above four ſides is called, in general, a *polygon.*

18. A *circle* is a plain figure bounded by one continued line called the *circumference*, which is every-where equally diſtant from a point within the circle, called the *center.*

19. Any line drawn from the center to the circumference is called a *radius*; the double of which is the *diameter* of the circle.

20. The meaſure of an angle is the arc deſcribed from the angular point as center with any radius, and terminated by the lines which form the angle.

21. A line which touches a circle in one point only, is called a *tangent*; and the point where it touches, the point of *contact.*

22. A polygon is called *regular*, when all its ſides are equal as well as all its angles; and *irregular*, when they are not equal.

23. A polygon is ſaid to be *inſcribed* in a circle, when all its angles touch the circumference; and circumſcribed about a circle, when all its ſides touch the circle.

Polygons are diſtinguiſhed by the number of their ſides, viz.

5 Pentagon,
6 Exagon,
7 Eptagon,
8 Octagon,
9 Enneagon,
10 Decagon,
11 Ondecagon,
12 Dodecagon.

### PROBLEM 1.

*To divide a given right line into two equal parts.*

Let the line given be AB, Plate I. Fig. 3. Set one foot of the *compaſſes* in A, and with any convenient diſtance ſweep two *arches* one above, and the other below, the given *line*; then ſet one foot of the *compaſſes* in B, and with the ſame diſtance as before croſs the aforeſaid *arches*, join the points of interſection with a right line, and it will cut AB in two equal parts.

### PROBLEM 2.

*To raiſe a perpendicular from any point propoſed, in a given right line.*

Let the given line be AB, Plate I. Fig. 4. and the *perpendicular* to be raiſed from the point C; to do which, ſet off two equal diſtances from the point C, as CR, CS: then the *compaſſes* being opened to any convenient diſtance greater than CR or CS, with one foot of the *compaſſes* in the point R deſcribe the *arch* DE: then with the ſame extent, and one foot of the *compaſſes* placed in the point S, deſcribe the *arch* FG; then draw from C a right line through the interſection of the two arches FG, DE, and it will be the *perpendicular* required.

### PROBLEM 3.

*To let fall a perpendicular from a point aſſigned, to a given right line.*

Let AB, Plate I. Fig. 5. be a right line given, and C the point from whence the *perpendicular* muſt fall.

Set one foot of the *compaſſes* in C, and opening them to a proper diſtance deſcribe the arch RS; then by Prob. 1. divide RS into two equal parts in D, join the points C, D, and CD will be the *perpendicular* required.

### PROBLEM 4.

*To draw a right line parallel to a given right line, and alſo through a point given.*

Let AB, Plate I. Fig. 6. be the given line, and C the point given. From C let fall, by the laſt Problem, the perpendicular CD upon the line AB. Take any point, as E in AB, in which ſet one foot of the *compaſſes*, and with an opening equal to CD, deſcribe an *arch* FG: lay a ruler, ſo as juſt to touch this *arch* and the point C, then draw the line CH, it will be the *parallel* required.

### PROBLEM 5.

*To divide a given right line into any number of equal parts.*

Let AB, Plate I. Fig. 7. be the given line.

From the point A draw a right line AC at pleaſure; from the point B draw a right line BD parallel to AC, ſet off along AC and BD from the points A and B, as many equal parts AE, EF, FG, &c. and BK, KL, LM, MN, &c. towards C and D reſpectively, as are one leſs than the number into which you intend the given line to be divided; join the laſt point in AC with the firſt in BD, and alſo join the other points I, L, G, N, F, O, &c. and the right lines KK, IL, HM, GN, &c. will divide AB into the deſired number of equal parts at the points k, i, h, g, &c.

### PROBLEM 6.

*To divide a given angle* ACB, Plate I. Fig. 8. *into two equal parts.*

From the angular point C as center, deſcribe an arch with any radius, meeting the legs of the angle at A, B: from theſe points, as centers, deſcribe two *arches* with the ſame radius, ſo as to interſect each other at D; then the line drawn through this interſection and the angular point C, will biſect the given angle.

### PROBLEM 7.

*To draw a line* CA, *ſo as to make an angle* ACB, *with a given line* CB, *equal to a given angle* acb, Plate I. Fig. 9.

From the points C, c, as centers, deſcribe two arches with any radius at pleaſure: make the arch AB equal to the arch ab; and the line drawn through the points C, A, will form the angle required.

Note, The circumference of every circle, whether great or ſmall, is ſuppoſed divided into 360 equal parts, called degrees, each degree into 60 equal parts, called minutes. Hence a quadrant, or the meaſure of a right angle, is 90 degrees, being the fourth part of the circumference; a ſemicircle, or the meaſure of two right angles, is 180 degrees, being the one half of the circumference.

### PROBLEM 8.

*To deſcribe the circumference of a circle, through three given points* A, B, C, Plate I. Fig. 10.

Join theſe points by two lines AB, BC, biſect them with the perpendiculars DO, EO; their point of interſection O, will be the center of the circle deſired.

### PROBLEM 9.

*To inſcribe a ſquare in a circle.* Plate I. Fig. 11.

Draw the diameter AB, to which draw the diameter CD perpendicular; then the lines joining the extremities of theſe diameters will be the ſquare ABCD required.

### PROBLEM 10.

*To inſcribe a regular polygon of a given number of ſides in a circle.*

Draw two *radii* from the centre of the propoſed circle, making an angle equal to the *angle at the centre of a polygon* anſwering to the aſſigned number of ſides (as in the table annexed): join the two points in the circumference, where the *radii* interſect it, and that line or *chord* will be the ſide of the required polygon.

## A TABLE of Regular POLYGONS.

| Polygon. | Angle at the Center. | Angle made by the Sides. |
|---|---|---|
| 5 | 72 | 108 |
| 6 | 60 | 120 |
| 7 | $51\frac{3}{7}$ | $128\frac{4}{7}$ |
| 8 | 45 | 135 |
| 9 | 40 | 140 |
| 10 | 36 | 144 |
| 11 | $32\frac{8}{11}$ | $147\frac{3}{11}$ |
| 12 | 30 | 150 |

The angle at the centre of a regular polygon is found by dividing 360 degrees by the number of the ſides: thus 360 divided by 5 gives 72 degrees for the angle at the center of a Pentagon; and 360 divided by 6 gives 60 degrees for the angle at the centre of an Exagon.

But the angle made by two adjacent ſides of a polygon is found by ſubtracting the angle of the centre from 180 degrees: thus from 180 degrees take 72, there remains 108 degrees, which is the angle made by the ſides of a Pentagon; and if from 180 we take 60 (the angle at the center of an Exagon), there will remain 120, the angle made by the ſides of an Exagon: in the ſame manner may the remaining numbers in the third column of the foregoing table be found.

Having furniſhed the reader with the moſt neceſſary geometrical Problems, for the underſtanding of Perſpective, we ſhall next proceed to the definition thereof.

DEFINITION

### DEFINITION I.

In order to have a clear idea of the principles of this art, we are to conſider that a picture drawn perfectly true, and placed in a proper poſition, ought ſo to appear to the ſpectator, that he ſhould not be able to diſtinguiſh the repreſentation from the real original objects actually placed where they are repreſented to be. To produce this effect, it is neceſſary that the rays of light ought to come from the ſeveral parts of the picture to the ſpectator's eye with the ſame circumſtances of direction, ſtrength of light and ſhadow, and colour, as they would do from the correſponding parts of the real objects ſeen in their proper places. Thus, Plate II. Fig. 1. let O be the ſpectator's eye, FH the plane of the picture, which, for more eaſy conception, we may ſuppoſe to be tranſparent, ABCDE an original cube; then if we imagine lines to be drawn from O to every part (in view) of the ſolid ABCDE, their interſection with the plane FH will mark thereon the repreſentation abcde of the original figure ABCDE.

2. *Cone of rays.* Lines drawn from the ſeveral parts of any figure, and continued ſo as to paſs through one and the ſame point, conſtitute a cone of rays; and when that point is conſidered as the eye of a ſpectator, it is called the *optic cone*.

3. *Ichnography.* When a ſyſtem of rays (parallel to each other and perpendicular to the horizon) coming from the ſeveral points in any figure, is cut by a plane parallel to the horizon, the projection thereon is called the *ichnography of the ſaid figure*, as Fig. 2. Plate II.

4. *Orthography.* When a ſyſtem of rays (parallel to each other and to the horizon) coming from the ſeveral points in any figure, is cut by a plane perpendicular to the horizon, the projection thereon is called the *orthography of the ſaid figure*, as Fig. 3. Plate II.

Theſe are the *common* definitions of the terms *ichnography* and *orthography*; but in the following pages we ſhall uſe them to ſignify any two projections that are made by ſyſtems of parallel rays, when thoſe ſyſtems are perpendicular to each other, and to the planes on which the projections are made, without having any regard to their ſituation with reſpect to the horizon.

5. *Schenography.* When the optic cone is cut by a plane, the repreſentation of the propoſed figure thereon is called the *ſchenography* thereof. It is evident that the ſhadows of figures are this projection, when the light is conſidered as a ſingle point: though in the caſe of the ſun or moon, that point being at an infinite diſtance (as to all ſenſe), the projecting rays are parallel to each other.

6. *Point of ſight* is the vertex of the optic cone, or it is that point where the ſpectator's eye ſhould be placed to look upon the picture.

7. *Diſtance of the picture.* The point in which a perpendicular let fall from the *point of ſight* upon the picture cuts it, is called the *center* of the picture; the diſtance between the *center* and *point of ſight* is called the *diſtance of the picture.*

8. *Directing plane* is that plane which paſſes through the *point of ſight* parallel to the picture.

9. *Original object*; the real object (whether it be point, line, ſurface, or ſolid), placed in the ſituation it is repreſented to have by the picture.

10. *Original plane.* The plane wherein is ſituated any original point, line, or plain figure, is called the *original plane.*

11. *Interſection of an original line* is that point wherein the ſaid line (continued if need be) cuts the picture.

12. *Interſection of an original plane* is that line wherein the ſaid plane cuts the picture.

13. *Directing point of an original line* is that point wherein the ſaid line cuts the *directing plane.* And a right line joining the directing point and point of ſight, is called the *director* of that original line.

14. *Directing line of an original plane* is that line wherein an original plane cuts the directing plane.

15. *Parallel of an original line* is a right line drawn through the point of ſight parallel to the ſaid original line.

16. *Parallel of an original plane* is a plane paſſing through the point of ſight parallel to the ſaid original plane.

17. *Vaniſhing point* is the point where the parallel of any original line meets the picture, and is called the *vaniſhing point* of that line; the diſtance between the vaniſhing point and the point of ſight is called the *diſtance* of that vaniſhing point.

18. *Vaniſhing line* is that line wherein the parallel of any original plane interſects the picture. From the point of ſight let fall a perpendicular to the vaniſhing line; the point where that vaniſhing line is ſo cut is called the *center* of it, and the diſtance between the center and point of ſight is called the *diſtance* of that vaniſhing line.

19. *Repreſentation of any figure* is the projection of that figure. In order to comprehend the ſenſe of theſe definitions more fully, let the planes ABIC, DEO, Plate III. Fig. 1. be ſo raiſed up as to ſtand parallel to each other, and bring the plane ACO parallel to the plane FGH: this done, imagine the plane ABIC to be the ſurface of the picture, O to be the point of ſight, (Def. 6. Perſpective) the plane DEO to be the directing plane, (Def. 8.) FG to be an original line (Def. 9.) in the original plane FGH, (Def. 10.) cutting the picture in BI and the directing plane in DE: and let ACO cut the picture in the line AC, and ſuppoſe OV to be drawn parallel to the original line FG, and cut the picture in V: and let FG (produced) cut the picture in B, and the directing plane in D; then is B the interſection of the original line FG, (Def. 11.) D its directing point, (Def. 13.) and V its vaniſhing point, (Def. 17.) and OD its director, (Def. 13.) and OV is the diſtance of the vaniſhing point V (Def. 17.); BI is the interſection, (Def. 12.) DE the directing line, (Def. 14.) ACO the parallel, (Def. 16.) and AC the vaniſhing line (Def. 18.) of the original plane FGH. From O let fall a line perpendicular upon the vaniſhing line AC, and the point wherein the ſaid perpendicular meets AC, as S, will be the *center* of the picture, (Def. 18.) and SO will be the diſtance of the vaniſhing line AC (Def. 18.)

### AXIOM I.

The common interſection of the two planes is a ſtraight line.

### AXIOM 2.

If two ſtraight lines meet in a point, or are parallel to one another, there may be a plane paſſing through them both.

### AXIOM 3.

If two parallel ſtraight lines are cut by a third line, they will all three be in the ſame plane; that is, a plane paſſing through any two of them will alſo paſs through a third.

### AXIOM 4.

Every point in any ſtraight line is in any plane that line is in.

### THEOREM I.

A line drawn from the center of the picture to the center of a vaniſhing line, is perpendicular to that vaniſhing line.

### DEMONSTRATION.

Raiſe up the plane AC (Plate III. Fig. 2.) perpendicular to the plane of the picture AFG; draw OC perpendicular to AF, and move the triangle OSC about OC as an axis, until its baſe CS is perpendicular to a given vaniſhing line, as AB; then will OS be perpendicular to ASB, and conſequently (Def. 18) S is the center of the vaniſhing line propoſed.

### COROLLARY.

The diſtance OS of any vaniſhing line ASB, is the hypothenuſe of a right angled triangle, whoſe legs are the diſtance of the picture OC, and the diſtance CS between the center of that vaniſhing line and the center of the picture. Thus Fig. 1. in Plate IV. is the perſpective repreſentation of the ſeveral planes which are perpendicular to one another, and anſwers to this corollary. O is the place of the ſpectator's eye in the directing plane, C the center of the picture, and ASB a vaniſhing line.

THEOREM

### THEOREM 2.

The perſpective repreſentation, or projection of any object, is the ſame as the ichnographic projection of it on the plane of the picture, the point of ſight being the vertex of the optic cone.

### DEMONSTRATION.

For by the explanation of the principles in Definition 1. the light muſt come to the ſpectator's eye O (Plate II. Fig 1.) in the ſame direction from any point *a* of the projection as it would do from the correſponding point A of the original object; therefore it is evident that the rays *a*O and AO are in one and the ſame ſtraight line. Whence it follows that the projection *a* is the interſection of the Picture with the ray AO, and the whole projection *abcde*, is the ſchenography of the original ABCDE, made by the optic cone OABCDE, whoſe vertex is the point of ſight O.

### COROLLARY 1.

The projection of a ſtraight line is a ſtraight line. For conceive a plane paſſing through the point of ſight, and the original line to interſect the picture, it is then evident that the interſection can only be a ſtraight line. As *de* is the interſection of the picture with the triangle ODE, and conſequently a ſtraight line (by Ax. 1.)

### COROLLARY 2.

The original of a projection may be any object that will produce the ſame cone of rays. Thus the original of the projection *de* (Fig. 2. Plate IV.) may be any line EF, which produces the optic cone ODE, as well as the line DE; for the plane OEF interſects the picture in *de*; therefore the projection *de* is the repreſentation of EF or the line DE. The Figure 2. Plate IV. explains Corollary 1. of this Theorem; the plane PG repreſents a picture perpendicular on the plane DH, where an original line DE is drawn, OK the directing plane, O the point of ſight, and ODE the plane paſſing through the point of ſight and the original line DE.

This being ſo, it may be reaſonably aſked, whence it happens that figures drawn on a picture appear to be what they are deſigned to repreſent. The reaſon is, becauſe the mind has acquired a habit of judging objects, that are ſo and ſo related, have ſuch and ſuch colours, and are ſo and ſo enlightened and ſhaded, to be of ſuch a ſhape and alike ſituated.

Theſe circumſtances are all of them neceſſary to make a picture compleat, though the ſimple drawing is ſometimes almoſt ſufficient, upon account of the relation of the parts; as in a pavement, where all the ſtones appear to be ſquare, though they are repreſented by very irregular figures. I ſay, it is the relation of the parts which produces this effect; for the repreſentation of any one of the ſingle ſquares would hardly appear to be ſquare, were there no objects to biaſs the judgment by their relation to it.

### THEOREM 3.

The projection of a ſtraight line not parallel to the picture, paſſes through both its interſection and vaniſhing point.

### DEMONSTRATION.

Imagine an indefinite plane (paſſing through the point of ſight and the original line) to cut the picture, then it is very obvious that in the line which forms their interſection, that of the original line with the picture will always be found: but by Def. 17. the line determining the vaniſhing point will alſo be in this plane, therefore the projection of an original line F G (Plate III. Fig. 1.) paſſes through its interſection B and vaniſhing point V.

The ſame ſcheme repreſented in perſpective. (Fig. 3. Plate IV.) the plane C I is the picture perpendicular on the plane E H, and parallel to the directing plane O E, a plane O D K E paſſing the point of ſight O, and the original line F G cuts the picture in the interſecting point B, and vaniſhing point V; and the interſection of the plane O D K E with the picture will give the line B V, for the indefinite repreſentation of the line B G E. From any points G F draw the rays O G, O F cutting the picture in *g f* for the repreſentation of G F: for as the original line G F is in that plane, its repreſentation will be ſomewhere in BV. The projection *gf* being produced will meet the original line, (if produced to cut the picture) in its interſecting point B, and will alſo paſs through its vaniſhing point V, which is produced by its parallel OV. This theorem being the principal foundation of all the practice of *perſpective*, the learner would do well to make it very familiar to him. In order to which I have again repreſented the meaning of it in Plate II. Fig. 1. where the projection *bc* meets the original line BC in its interſection K, and paſſes alſo through its vaniſhing point V, which is produced by its parallel OV.

N. B. When the original line (CK, Plate IV. Fig. 1.) itſelf paſſes through its vaniſhing point, the whole projection of it will be that point; ſo that in that caſe the line may be ſaid to vaniſh: this is one reaſon for uſing that term. Another reaſon is, the further any object is off upon any line, the ſmaller is its projection, and, at the ſame time, the nearer to this point; and when it comes into this point, its magnitude will entirely vaniſh, becauſe the original object is at an infinite diſtance. This is eaſily conceived by imagining a perſon going from you in a long walk, who appears to be ſmaller and ſmaller the farther he goes. The reaſon of this diminution will appear on examining the ſcheme, Fig. 4. Plate IV. where the parallel lines DB and C~~D~~A repreſent two rows of trees (or a long walk), PP the ſection of a picture, O the place of the ſpectator's eye, *dc* the repreſentation of the trees DC, which is evident as theſe points are in the lines drawn from DC to the eye. As the trees DC are nearer the eye of the ſpectator than any other between the lines DB and CA, the angle it makes with the eye is larger, therefore its appearance to the ſpectators will be greater; but the trees BA being farther off, the lines proceeding from them to the eye will form a ſmaller angle, and cut the picture in a much leſs ſpace, as *ba*, which is the reaſon of its appearing ſmaller than the original of *dc*; and the farther any object is remote from the eye the ſmaller its appearance will be, ſo that the rows of trees DB, CA, might be continued ſo far that the ſpace between BA would only be a point in the picture, in that caſe it may be ſaid to vaniſh.

COROLLARY

### COROLLARY 1.

Projections of original lines that are parallel to each other, but not to the picture, paſs through the ſame vaniſhing point: for they have but one parallel common to them all, and conſequently but one vaniſhing point.

As in Plate II. Fig. 1. where the projection *da* and *cb* of the parallel lines DA and CB meet in their common vaniſhing point V. And in Plate IV. Fig. 1. the projection DC and EC of the parallel lines DG and EF meet in their common vaniſhing point C.

### COROLLARY 2.

The center of the picture is the vaniſhing point of lines perpendicular to the picture.

As in Plate IV. Fig. 1. OC being drawn perpendicular to the picture, C is the center (by Def. 7.) and is alſo the vaniſhing point of the lines DG and EF, they being perpendicular to the picture.

### THEOREM 4.

The projection of an original line parallel to the picture is alſo parallel to that original line.

### DEMONSTRATION.

Imagine a triangular plane, whoſe vertex is at the ſpectator's eye, and its baſe the propoſed original line, to be cut by the picture. It is evident the repreſentation upon the picture will be parallel to the baſe of this triangular plane, becauſe the picture is ſuppoſed to be ſo; therefore if EF, Plate III. Fig. 3. be the picture, AB an original line parallel to it, then will its projection *ab* be parallel to the original AB, O being the point of ſight.

### COROLLARY 1.

Lines parallel to one another, and to the picture, are projected into lines parallel among themſelves. Thus *ab* and *dc* are parallel to each other, and to their originals AB and CD.

### COROLLARY 2.

The projection *abcd* of any plain figure ABCD parallel to the picture is ſimilar to its original. For draw AC and *ac*, then will the triangles ACB, *acb*, be ſimilar, as will alſo the triangles ADC, *adc*, conſequently ABCD and *abcd* are ſimilar.

### COROLLARY 3.

AB is to *ab* as the diſtance of the picture is to the diſtance between the point of ſight and the plane of the original figure; that is, (O*g*G being drawn perpendicular to thoſe two planes) AB:*ab*::O*g*:OG.

### THEOREM 5.

The projection of a line is parallel to its director. For the lines OF, OG, OD, *fg*, (Plate III. Fig. 1.) are all in the ſame plane; but the directing plane ODE is parallel to the plane of the picture ABIC (Def. 8.) therefore the director OD is parallel to the projection *fg*. Euclid, II. 16.

### COROLLARY 1.

The projections of lines that have the ſame director, are parallel to each other.

As in Plate IV. Fig. 5. OD is the director of the original lines AL, EF, (by Def. 13.) and *ai*, or *ef*, their projection on the picture GH, are parallel to one another.

### COROLLARY 2.

If the original line is parallel to the picture, its director is ſo too, and conſequently is in the parallel of any plane paſſing through that original line; and therefore the vaniſhing line of that plane and the projection of the line are parallel to one another.

As in Plate IV. Fig. 6. GK an original line parallel to the picture, *ab* its projection parallel to the vaniſhing line of the plane H, which the original line is on.

### THEOREM 6.

The vaniſhing line, interſection, and directing line of any original plane, are parallel to each other.

For the planes OVC, DFH, (Plate III. Fig. 1.) are parallels (Def. 16) as alſo ODE and CAB (Def. 8.) therefore the vaniſhing line CV, interſection IB, directing line ED, are parallel to each other. The Fig 6. Plate IV. anſwers to this.

### COROLLARY.

The triangles of V, DOF, are ſimilar, and DO=BV, therefore *f*V:VO::BV:DF.

### THEOREM. 7.

All lines in any original plane have their vaniſhing points in the vaniſhing line of that plane.

### DEMONSTRATION.

For as all the original lines are in the ſame plane, the lines which determine their vaniſhing points, will interſect the picture in the vaniſhing line of the original plane.

### COROLLARY 1.

Original planes parallel to each other, have the ſame vaniſhing line.

### COROLLARY 2.

The vaniſhing point of the common interſection of two original planes, is the common interſection of their vaniſhing lines.

As in Plate IV. Fig. 7. the point L is the vaniſhing point of the common interſection of the two original planes, (PRIF and RNZI) being the common interſection of their two vaniſhing lines LH and LV.

### COROLLARY 3.

The vaniſhing line of a plane perpendicular to the picture, paſſes through the center of the picture.

As in Plate IV. Fig. 1. CD is the vaniſhing line of the plane EFH, paſſing through C.

### THEOREM 8.

Interſections of all lines in the ſame original plane, and alſo the line which determines the interſection of that original plane with the picture, are in that plane.

This needs no demonſtration.

### COROLLARY 1.

The interſection of two original planes which are not parallel to the picture, being produced, will interſect the picture in that point which is the common interſection of the two planes therewith.

As in Plate IV. Fig. 7. O, being the point of ſight, the planes FIP and ZRN, interſect the picture K in the point R, and RP is the interſection of the plane FIP with the picture, HL its vaniſhing line (by Def. 18.) RN is the interſection of the inclined plane ZRN, with the picture, LV its vaniſhing line (by Def. 18.) and the point L the vaniſhing point of the common interſection of the original planes, being the interſection of their vaniſhing lines; it follows that RL is the repreſentation of RI, the interſection of the original planes, on the picture.

### COROLLARY 2.

If the common interſection of ſeveral planes is parallel to the picture, then thoſe planes have parallel interſections therewith, and have alſo parallel vaniſhing lines.

### PROBLEM I.

Having given the center and diſtance of the picture, to find the projection of a point, whoſe ſeat* on the picture, with its diſtance from it, are given.

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* By *ſeat on the picture*, we mean that point wherein a perpendicular falling on the picture from the point to be projected, interſects it.

Let O be the ſpectator's eye, Plate III. Fig. 4. S the center of the picture, and $b$ the given ſeat of the original point A; then if we imagine a line drawn from O to A, it will cut the plane of the picture ſomewhere in that line which joins the center of the picture and ſeat of the original line, as at $a$, which is therefore the repreſentation of the point A, by Theo. 2. the triangles $Aab$ $OaS$ are ſimilar; whence $Sa:ab::SO:Ab$.

### COROLLARY. 1.

The point A may be found by a ſcale and compaſſes, dividing the line $Sb$ in $a$, ſo that $Sa$ may be to $ab$, as the diſtance of the picture OS, is to the diſtance $Ab$ of the original point from its ſeat $b$.

### COROLLARY 2.

By this Problem the projection of any right line may be found; for it is no more than finding the projection of any two points therein, and drawing a right line through thoſe two projections.

### PROBLEM II.

To find the projection of a line, its vaniſhing point and diſtance; having given its ſeat, interſection, and the angle it makes with its ſeat, and the center and diſtance of the picture.

Let W (Plate V. Fig. 1.) be a point in the given line, E its ſeat on the picture, and D its interſection.

Make SO equal to the diſtance of the picture, draw SV parallel to DE and perpendicular to SO, join the center of the picture and ſeat of the point W by drawing ES; then it is evident that the projection of W will be in this line: but to determine the exact place, we muſt divide ES in the proportion of WE to SO, which may be done thus: Make $ET=EW$, $SM=SO$: draw TM, and the point of interſection $w$ will be the place required: join the points D, $w$, and $Dw$ will be the indefinite repreſentation of the line propoſed. If from the point of ſight, a right line be drawn parallel to DW, it will interſect the picture in the vaniſhing point thereof, and its diſtance will be equal to OV.

### COROLLARY 2.

If the vaniſhing point and interſection of the original line $DW$ be joined by a right line, it will cut $ES$ in $w$, the projection of the original point $W$.

### PROBLEM III.

Having given the projection of a line, and its vaniſhing point, to find the projection of the point that divides the original line in any given proportion.

Let $AB$ (Plate V. Fig. 2.) be the given projection of the line to be divided, and $V$ its vaniſhing point. Draw at pleaſure $VO$ and $ba$ parallel to it; and through any point $V$ of the line $VO$, draw $OA$ and $OB$ cutting $ba$ in $a$ and $b$. Divide $ab$ in $c$ in the proportion given, and draw $Oc$ cutting $AB$ in $C$. Then ſhall $C$ be the projection ſought, the original of $BC$ being to the original of $CA$, as $bc$ is to $ca$.

For $OV$ being parallel to $ba$, $ba$ may be conſidered as the original line, and $OV$ as its parallel, and conſequently $O$ as the point of ſight, and $aO$, $bO$, $cO$, as viſual rays projecting the points $A$, $B$, $C$.

### COROLLARY.

$BV \times CA$ is to $CB \times AV$ as $ca$ is to $cb$. In order to prove this, draw $ef$ through $C$ parallel to $ab$, then it is evident that the triangle $ACe$ is ſimilar to the triangle $AOV$; whence we have $AC:AV::eC:OV$, and again the triangle $VOB$ is ſimilar to the triangle $fBC$, whence we get $BV:CB::OV:Cf$.

But before we had $AC:AV::eC:OV$, and conſequently $BV \times AC:CB \times AV::OV \times eC:Cf \times OV$; becauſe if four quantities are proportional, and any other four quantities are proportional, their product, taken two by two, will alſo be proportional, and therefore we have

$AC \times BV:AV \times CB::eC:Cf$, or $ca:cb$. Wherefore the point $C$ may be found by a ſcale and compaſſes, making

$CA:CB::AV \times ca:BV \times cb$. Which was to be proved.

Then by a ſcale of equal parts, the ratio as 4 to 5.

$$\begin{array}{lll}
 & \text{Pt. Dec.} & \\
\text{Suppoſe} & AB = 4.53 & CA : CB :: 9.3 \times 4 : 4.74 \times 5 \\
 & AV = 9.\ 3 & CA + CB : CB :: 37.2 + 23.70 : 23.70 \\
 & BV = 4.74 & \text{that is } 4.53 : CB :: 60.90 : 23.70. \\
\hline
\text{Found} & CB = 1.76 & \begin{array}{r} 4.53 \\ \hline 7110 \\ 11850\phantom{0} \\ 9480\phantom{00} \\ \hline 60.90)107.3610(1.76 = CB \text{ line ſought.} \end{array}
\end{array}$$

### PROBLEM IV.

Having the projection of a line, and its vaniſhing point, from a given point in that projection, to cut off a ſegment, that ſhall be the projection of a given part of the original of the projection given.

Let AB (Plate V. Fig. 3.) be the projection given, V its vaniſhing point, and C the point from whence is to be cut off the ſegment. Draw at pleaſure VO, *abc* parallel to it, and from any point O in VO, draw OA, OB, OC, cutting *ab* in *a*, *b*, *c*. Make *cd* to *ab*, as the given part is to the original of AB, and draw O*d* cutting AB in D: then will CD be the ſegment ſought.

The Demonſtration of this problem is obvious from that of the foregoing one.

### COROLLARY.

The point D may be found by a ſcale and compaſſes, making $DC : DV :: dc \times AB \times CV : ab \times AV \times BV$. For, per laſt Prob.

$$\left.\begin{array}{l} AB \times BV : AV \times BC :: ab : bc. \\ BC \times DV : BV \times CD :: bc : dc. \end{array}\right\} \text{whence } \begin{array}{l} AB \times CV \times DV \times dc = \\ AV \times BV \times CD \times ab. \end{array}$$

And therefore $DC : DV :: dc \times AB \times CV : ab \times AV \times BV$.

N. B. The foregoing Problem may be conſidered as a particular caſe of this, *viz.* when the point C in this coincides with one of the points A or B.

### PROBLEM V.

Having given the center and diſtance of the picture, to find the vaniſhing line (with its center and diſtance) of a plane, whoſe interſection is given, with the angle of its inclination to the picture.

Let C (Plate III. Fig. 2.) be the center of the picture. Draw CG parallel to the interſection of the original plane with the picture, and equal to the diſtance of the picture, and from C erect the indefinite right line CS perpendicular to CG; draw GS, making an angle SGC equal to the complement of the angle of inclination of the original plane to that of the picture; draw through S the right line ASB. Then will ASB be the vaniſhing line ſought, S its center, and GS its diſtance.

Becauſe GC = CO, and the angle GSC equal to the given inclination of the original plane and picture (by conſtruction), it is evident that the right-angled triangle GCS is equal to the moveable right-angled triangle COS; therefore, by Defin. 18. ASB is the vaniſhing line, and S its center.

N. B. Taking GC for radius, CS is the cotangent, and GS the coſecant of the inclination of the original plane to the picture.

### PROBLEM VI.

Having given the interſection of an original plane, with its vaniſhing line, its center and diſtance; to find the projection of any line in the original plane, having the original figures drawn out in their juſt proportions.

Let DF (Plate V. Fig. 4.) be the interſection given, HG the vaniſhing line, and S its center. Draw SO perpendicular to GH, and equal to the diſtance of the vaniſhing line GH, and let the ſpace X be the original plane, ſeen on the reverſe, as objects appear in a looking-glaſs; the ſpace Y being the parallel plane in the ſame manner folded down on the picture: and let AB be the original line whoſe projection is ſought. Let AB cut the interſection in D, and draw OG parallel to AB, cutting the vaniſhing line in G. Draw DG, which will be the indefinite projection of AB. Through A and B draw at pleaſure AC and BC meeting in C, and in the ſame manner find their indefinite projections FI, and EH, cutting DG in *a* and *b*. Then will *ab* be the determinate projection of AB, *a* being the projection of the extremity A, and *b* the projection of the extremity B.

OTHER-

### OTHERWISE,

Suppoſe KL to be the original line given. Having found its indefinite projection QG, as before, draw OK and OL, cutting it in *k* and *l*, which will be the projections of the extremities K and L.

### Otherwiſe, by the DIRECTORS.

Let DF (Plate V. Fig. 5.) be the interſection given. And let the original plane be folded down on the picture, ſo as to bring the directing line into the plane HI, the diſtance between AF and HI being equal to the diſtance of the vaniſhing line given. Let alſo O be the point of ſight brought into the picture at the ſame time along with the directing plane HIO. To find the indefinite projection of any original line AB, continue it till it cuts EF in F, and HI in G; then draw OG, and F*a* drawn parallel to it, will be the indefinite projection ſought. Then finding in the ſame manner the indefinite projection E*d* of any other line, AD paſſing through A, by its interſection with F*a*, is got the projection *a* of the extremity A. And in the ſame manner is got the other extremity *b*; or thoſe extremities might be found by drawing lines from A, and from B to O, as in the foregoing conſtruction.

To prove the truth of theſe operations, imagine the figures (Plate V. Fig. 4.) to be folded in DF and HG, and (Plate V. Fig. 5.) in EF and HI, till the original plane, its parallel, and the directing plane, and along with them the point of ſight O, come into their proper places. Then you will find, that D (Plate V. Fig. 4.) and F (Plate V. Fig. 5) will be the interſection of AB, and G (Plate V. Fig. 5.) will be its directing point. But OG (Fig. 4.) is ſtill parallel to AB, wherefore G is its vaniſhing point, and DG its indefinite projection, (by Theo. 3.) and F*a* (in Fig. 5.) is ſtill parallel to OG, which is the director of AB; wherefore F*a* is the indefinite projection of AB (Theo. 5.) That *a* found by the interſection of GI with DG (in Fig. 4.) and of E*d* with F*a* (in Fig. 5.) is the projection of the interſection of the original lines AB and AC (Fig. 4.) and of AB and AD (Fig. 5.) is obvious. The other conſtruction by the lines AO is the ſame as by the lines AO and CO for finding the points *a* and *c*, Problem II.

N. B. In Fig. 5. the projections *ad* and *kl* are parallel, their originals, having both of them the ſame director OH, according to Cor. 1. Theor. 5. The ſame may be obſerved in *lm* and *cd*, which have the ſame director OI.

### PROBLEM VII.

Having given the ſame things as in the foregoing problem, to find the projection of any figure in the original plane.

This is done by finding the projections of the ſeveral parts of the figure given, by the foregoing problem.

For example, the projection *klmnp* (Plate VI. Fig. 1.) of the pentagon KLMNP, is found thus. Drawing OG, OH, OI, OV, parallel to KL, LM, MN, KP reſpectively, the points G, H, I, and V, are vaniſhing points: and KL, LM, MN, being continued, cut the interſections in their interſections Q, R, T; whence drawing QG, RH, TI, are got the projections *l*, *m*, of the points L and M, by their mutual interſections. Then drawing OK and ON, are got the points *k* and *n*. Then drawing *k*V to the vaniſhing point V of KP, is got the indefinite projection of KP. Laſtly, drawing OP, is got the point *p*.

The projections of curve-lined figures are to be got by finding the projections of ſeveral of their points, and afterwards joining them neatly by hand. Thus (Plate VI. Fig. 2.) DE being the interſection, and VF the vaniſhing line, and O the point of ſight, and ABC an original circle, placed as in the foregoing problem, the projection *a* of any point A may be found by drawing at pleaſure AD, and OV parallel to it; then drawing DV and OA meeting in the point ſought *a*, according to the conſtruction in Problem VI. D being the interſection, and V the vaniſhing point of the line AD, and the ſeveral lines AD being drawn parallel to one another, the ſame vaniſhing point V may ſerve for them all.

Or VF (Plate VI. Fig. 3.) being the directing line brought into the picture, the reſt remaining as before, drawing at pleaſure AD cutting DE and VE in D and V; then drawing OV and D*a* parallel to it, the projection *a* is got by drawing OA cutting D*a* in *a*. And the ſame point V being uſed for all the points A, all the lines D*a* will be parallel to one another, and to the ſame line OV.

## PROBLEM VIII.

To find the projection of any figure in a plane parallel to the picture.

The projection being ſimilar to its original, (by Cor. 2. Theo. 4.) this is done by making an exact copy of the original figure; making the homologous ſides in the proportion explained in Cor. 3. of the ſame Theorem.

## PROBLEM IX.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of a line in that plane; to find the projection of another line in that plane, making a given angle with the former.

Let the plane HFIE (Plate VI. Fig. 4.) repreſent the picture, which raiſe up and place the plane EIO parallel to the original plane HBFH. Now if we ſuppoſe O to be the point of ſight, EI will repreſent the vaniſhing line. Let *ab* be the given projection; it is required to draw *ac*, ſo that the original of the angle *bac* may be equal to a given angle. Continue *ab* to its vaniſhing point G, draw GO and OI, making GOI equal to the given angle, and cutting the vaniſhing line in I; then draw I*ca*, which will be the line ſought.

The reaſon of this conſtruction is very eaſy to be underſtood; for the plane EOI being parallel to the original plane, it is evident that any two lines, as EO, GO, drawn from the vaniſhing points E, G, to the point of ſight O, will form an angle at that point equal to the angle contained by thoſe original lines, of which E and G are the vaniſhing points.

N. B. If it had been required to make *abc* to repreſent the angle ABC, the angle EOI muſt have been made equal to the complement of the angle ABC to two right angles.

### PROBLEM X.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of one ſide of a triangle of a given ſpecies in that plane; to find the projection of the whole triangle.

The projections of the ſides wanting are to be found by the foregoing problem, the angles of the triangle being given. Thus having given the projection *ab*, (ſee the laſt Figure) of the ſide AB of the triangle ABC, the vaniſhing point of the ſide *ac* is found by making the angle IOG equal to the angle CAB, and the vaniſhing point of the ſide *bc* is found by making the angle IOE equal to the complement of the angle CBA to two right angles.

N. B. If the vaniſhing point of the line given *ab*, is out of reach, you may proceed thus: Taking any line DR (Plate VII. Fig. 1. (parallel to the vaniſhing line HG by Theorem 6.) for the interſection, by means of two lines, H*b*E, I*a*F, drawn at pleaſure through *b* and *a*, find the originals A and B of the points *a* and *b*; (by the converſe of Problem I.) and draw AB. Then on the ſide AB complete the original triangle, and find the projection of the ſides wanting by Problem VII.

### PROBLEM XI.

Having given the vaniſhing line of a plane, its center and diſtance, and the projection of one ſide of any figure in that plane, to find the projection of the whole figure.

Divide the whole figure given into triangles, by means of diagonals, and find the projections of theſe triangles one after the other (by the laſt Problem), beginning with thoſe that have the line given for one of their ſides.

## PROBLEM XII.

Having given the center and diſtance of the picture, and the vaniſhing line of a plane, to find the vaniſhing point of lines perpendicular to that plane.

Let AB (Plate VII. Fig. 2.) be the vaniſhing line given, and C the center of the picture. Draw CA perpendicular to AB, and CO parallel to it, and equal to the diſtance of the picture. Draw AO and OD perpendicular to it, cutting CA in D, which will be the vaniſhing point ſought; for raiſe up the triangle AOD perpendicular to the plane of the ſcheme, (which here repreſents the picture) then a plane paſſing through the point of ſight O, and the line AB will be the parallel of the original plane, and the line OD will be perpendicular to it, and conſequently will be the parallel of lines perpendicular to that original plane. Wherefore D is the vaniſhing point of thoſe perpendiculars, (by Def. 17.)

N. B. 1. When the vaniſhing line AB paſſes through the center of the picture, that is, when the original plane is perpendicular to the picture, the point D will be infinitely diſtant, the line OD being the parallel to AD, and the projections of the lines perpendicular to the plane propoſed, will all of them be perpendicular to AB, they being to meet the line AC, which is perpendicular to it at an infinite diſtance, and conſequently they will be parallel to one another; which they ought to be upon another account, their originals being all parallel to the picture.

N. B. 2. But when the original plane is parallel to the picture, the diſtance CA will be infinite, and conſequently OA will be parallel to CA, and OD will conincide with OC, making the point D to fall into the center of the picture C, agreeable to Cor. 2. Theo. 4.

N. B. 3. CD is a third proportional to AC and CO, as alſo is AD to AC and AO.

N. B. 4. OD is the diſtance of the vaniſhing point D.

## PROBLEM XIII.

Having given the center and diſtance of the picture, to find the vaniſhing line, its center and diſtance, of planes that are perpendicular to thoſe lines that have a certain vaniſhing point.

The ſolution of this problem eaſily follows from that of the laſt; for let C be the center of the picture, and D the vaniſhing point propoſed, (ſee the laſt figure) then it is very evident that in order to determine the point A, we muſt join the points D C, and draw C O perpendicular to D C, and equal to the diſtance of the picture; then draw D O, and make O A perpendicular to it; draw A B perpendicular to the right line D C A, and it will be the vaniſhing line required, A being its center, (by Theo. 1.) and O A its diſtance.

## PROBLEM XIV.

Having given the center and diſtance of the picture, through a given point to draw the vaniſhing line of a plane that is perpendicular to another plane, whoſe vaniſhing line is given, and to find the center and diſtance of that vaniſhing line.

Let A B (the ſame Figure as before) be the vaniſhing line given, and C the center of the picture, and let E be the point given. Find the vaniſhing point D of lines perpendicular to the original plane of A B (by Prob. XII.): Draw D E, which will be the vaniſhing line ſought; draw C F cutting D E, at right angles in F, and F will be the center of the vaniſhing line D E (by Theo. 1.) Make a right-angled triangle, whoſe baſe is C F, and its perpendicular is equal to the diſtance of the picture, and its hypothenuſe will be the diſtance of the vaniſhing line D E (by the Cor. Th. 1.)

Now becauſe the plane, whoſe vaniſhing line is ſought, is perpendicular to the other plane, its vaniſhing line muſt paſs through the vaniſhing point D of lines perpendicular to that other plane, becauſe ſome of thoſe lines are in the plane ſought. Therefore D E is the vaniſhing line ſought.

In the above conſtruction, where DE is ſaid to be the vaniſhing line ſought, it is to be underſtood as the vaniſhing line of a plane perpendicular to that whoſe vaniſhing line AB is given, without having regard to its ſituation with reſpect to the picture, only that it ſhall be perpendicular to the plane of which AB is the vaniſhing line; for were the propoſed perpendicular plane, when produced, to interſect the picture in a given angle, ſuch reſtriction might render it impoſſible that the plane producing its vaniſhing line, ſhould paſs through both the points D, E.

### COROLLARY 1.

If FC be continued till it cuts the vaniſhing line given in B, B will be the vaniſhing point of lines perpendicular to the original plane of the vaniſhing line DE: for that vaniſhing point is in the line FC, by the conſtruction of Problem XII. and it is in the vaniſhing line given by what has been proved in the preſent Problem.

### COROLLARY 2.

And therefore, if the vaniſhing lines AB and DE meet in G, the points B, D, and G, will be the vaniſhing points of the three legs of the ſolid angle of a cube, which are perpendicular to one another. And drawing DB, GD, and GB will be the vaniſhing lines of the three planes that contain the ſolid angle.

### COROLLARY 3.

The diſtance of the vaniſhing line DG is equal to the line FP, the point P being the interſection of the line FC, with a circle deſcribed on the diameter DG.

### PROBLEM XV.

Having given the center and diſtance of the picture, and the vaniſhing point of the common interſection of two planes that are inclined to one another in a given angle, and the vaniſhing line of one of them; to find the vaniſhing line of the other of them.

Let C (Plate VII. Fig. 3.) be the center of the picture, BG the given vaniſhing line of one of the planes, and B the vaniſhing point of their common interſection, and H the angle of their inclination to one another. Find the vaniſhing line GD, of planes perpendicular to the lines whoſe vaniſhing point is B (by Prob. XIII.); let the vaniſhing line cut the vaniſhing line given in G. In GD find the vaniſhing point E of lines making the given angle H, with the lines whoſe vaniſhing point is G (by Prob. IX.); that is, in BCF perpendicular to GFD; take FP equal to the diſtance of the vaniſhing line GD, (found by Prob. XIII.) and draw PG, and PE, making the angle EPG equal to H; draw BE, which will be the vaniſhing line ſought.

For a demonſtration of this conſtruction, raiſe up the planes DPG, BPG, until they meet in P, the point of ſight perpendicular over the center of the picture C; then if we imagine a plane to paſs through P, and making the vaniſhing line BD, the planes BPG, GPD, DPB, will be the parallels of three original planes, whoſe vaniſhing lines are BG, GD, DB; that whoſe vaniſhing line is DG, being perpendicular to the other two (by the conſtruction, becauſe it is perpendicular to their common interſection, whoſe vaniſhing point is B.) Therefore the original planes, whoſe vaniſhing lines are BG and BE, are inclined to one another in the angle EPG, or H; (for the inclination of two planes is always meaſured in a plane perpendicular to their common interſection); therefore BG being the vaniſhing line given, BE is the vaniſhing line ſought.

N. B. The center of the vaniſhing line BE is found by drawing a line perpendicular to it from C, (by Theo. 1.) and then its diſtance is found, as was found the diſtance PF in Prob. XIV.

### OTHERWISE,

Having drawn the indefinite line BCF, make CF equal to a third proportional to BC, and the diſtance of the picture; draw DFG perpendicular to BF; then take FP equal to the quotient ariſing by dividing the ſum of the two ſquares of BC, and the diſtance of the picture, by the right line BF; draw GP, and from the point P draw PE, making an angle GPE equal to the given angle H: which being done, draw BE, and it will be the vaniſhing line required.

### PROBLEM XVI.

Having given the center and diſtance of the picture, and vaniſhing line of one face of any ſolid figure propoſed, and the projection of one line in that face; to find the projection of the whole figure.

By means of the projection given, find the projection of the ichnography of the figure propoſed on the plane of that face, whoſe vaniſhing line is given (by Prob. XI.); then (by Prob. XIV.) find the vaniſhing line of the plane of the orthography, and deſcribe the projection of the orthography, by help of the lines already given in the ichnography (by Prob. XI.) Laſtly, by the interſections of the projections of perpendiculars to the ichnography and orthography, will be found the ſeveral points of the projection required.

### OTHERWISE,

Having found the projection of the face whoſe vaniſhing line is given, by means of the projection of the lines given, find the vaniſhing lines of the adjacent faces, (by the laſt Problem) and deſcribe their projections by help of the lines given in the projection of the firſt face, and ſo on, till the whole projection ſought is completed.

In the practice of perſpective it is often neceſſary to draw lines towards a vaniſhing point, or ſome other point that is out of the picture, in ſuch caſes the following propoſitions are uſeful.

### PROBLEM XVII.

From a given point to draw a line that ſhall tend to an inacceſſible point of a given line, having the diſtance of the inacceſſible point, from a given point of that line.

Let Q, Fig. 1. Plate VIII. be the inacceſſible point in the line PQ, the diſtance BQ be given, and A is the point from whence a line is to be drawn tending towards Q, draw at pleaſure AP, cutting PQ in P, and taking the meaſure of PB by a ſcale of equal parts, you will have the length of PQ. Take any point *q* in the line BQ, and make P*a*:P*q*::PA:PQ. Draw *aq*, and AQ will be parallel to it. For ſuppoſe PQ equal to 15 parts, PA to 6 parts, and P*q* to 5, then it will ſtand thus.

As 6 : 15 :: 5 to P*a*.
6
15)30(2 = P*a*.

### PROBLEM XVIII.

Having given two lines that tend, to an inacceſſible point, though a given point to draw a line that ſhall tend to the ſame point. Let AB, CD, Fig. 2. Plate VIII. be the given line tending to the inacceſſible point, and P the point through which it is required to draw a line Pp tending to the ſame point. Through P draw at pleaſure two lines cutting AB and CB in A, C, B and D. Then draw AD and BC, meeting in Q. Through Q draw at pleaſure two lines cutting AB and CD in *b* and *c*, *a* and *d*, and draw *ac* and *bd* meeting in *p*, and Pp is the line required.

### PROBLEM XIX.

From any point in the picture, it is required to draw a line that ſhall tend to the vaniſhing point (which is out of the picture) of a given line, having the vaniſhing line of the plane the given line is on.

OQ (Fig. 3 Plate VIII.) is the vaniſhing line, PD the given line, and A the point through which the required line is to be drawn. Through A draw AB parallel to OQ at pleaſure, cutting PD in D. Make DB equal to AD; through D draw at pleaſure a line OD, cutting OQ in O; then draw BO ſo as to cut PD in any point *p*; through *p* draw *pa* parallel to BA, cutting DO in *a*; then draw A*a*, it will be the line required,

## PROBLEM XX.

Having given the diameter of a circle with the interſection of the plane (which the circle is on) on the picture. To find the vaniſhing line and diſtance, ſo that the repreſentation of the circle in the picture will be a circle.

Draw the line VB, (Plate IX. Fig. 2.) on it ſett off AB equal to the ſemidiameter of the given circle, and AV for the ſuppoſed diſtance of the picture always leſs than AB. Biſect BV in D, with the radius BD deſcribe the ſemicircle VCB, make BC equal to AB; draw CV for the ſpace between the interſecting and vaniſhing line.

Then let CB (Plate IX. Fig. 1.) be the interſecting line equal to twice AB (Fig 2.) Biſect it in A, and through that point raiſe the perpendicular AE; on it ſet off the line VC (Fig. 2.) and through V draw DD parallel to the interſecting line CB; make VE equal to VA in (Fig. 2.) and ſet it on DD; deſcribe the arches E*r*, E*s*, and DD is the vaniſhing line ſought, V its center, and VD or VE its diſtance.

To find the projection of the circle, draw D*a* B, D*a* C meeting in *a*; through *a* draw *cab* parallel to CB, and by drawing VB, VC, the points *b* and *c* will be found; through *c* or *b* draw a line to either of the points D, cutting VA in *d*; then through *c* draw *rc* 1, cutting the diagonal D*a*C in 1, and through *b* draw *sb* 2, cutting B *a*D in 2; and through *c* draw *cs*, cutting D*a* B in 4; and draw *rb* cutting D*a*C in 3; join theſe points for the projection of the circle. If you biſect A*d* in K and take KA for radius, it will inſcribe a circle paſſing through all thoſe points. The reaſon for this projection will appear clear from the next figure, where ABCD (Fig. 3. Plate IX.) is a plane containing the circle that touch the picture ABGH in the point L, ſo that the diameter LM of the circle is perpendicular to the picture. The point E is the ſpectator's eye, F its ſeat on the plane CDF, and EV the diſtance of the picture, (which can only be one determinated diſtance) to make the projection (1721 on the picture) of the circle ILMN, a circle alſo; make the angles EL7, EML equal, then conſequently the triangles E7L, and ELM will be equal, which are called ſubcontrary ſections.

If the ſcalene cone ABCD (Fig. 4. Plate IX.) be cut by a plane perpendicular to the baſe AC of the triangle of the axis ABC, in ſuch a manner that the triangle formed by the ſection EF (cutting the triangle ABC), be ſimiliar to the triangle ABC, but in a contrary poſition, which ſection is called ſubcontrary, that is, that the angle BEF be equal to the angle ACB, and the angle BFE to the angle BAC; the ſection EKF of the cone and the cutting plane, is a circle. The cutting plane may be ſuppoſed the picture.

But as there is a difference between the appearance and the projection of an object in the picture (as I ſhall ſhew hereafter,) the projection of that circle cannot appear a circle, unleſs the picture and the plane whichthe original circle is on, are parallel to one another, and the axes paſſing through the eye be perpendicular to the picture.

This is a general deſcription of the method to be uſed in putting any figures propoſed into Perſpective, and by the rules herein delivered, the following examples were actually delineated. Having in theſe few pages done my beſt endeavour to render Dr. Brook Taylor's Perſpective eaſy to be underſtood, I ſhould have here ended; but as ſeveral conſiderable writers upon this ſubject have objected againſt the univerſality of theſe rules, and their holding equally good, when applied to putting of ſome particular ſorts of objects, ſuch as round pillars, &c. into Perſpective, I hope the following attempt to ſet this matter in a clear light, will not be diſagreeable to the reader.

The appearance of an object and its repreſentation are two very different things, the former being determined by the angle (at the eye) under which it is ſeen; but the latter is the actual figure formed upon the picture, by the pencil, of rays paſſing from every point (in view) of the object to the ſpectator's eye; conſequently, in viewing a picture, if the eye be ſuppoſed to move about from the proper point of ſight deſigned by the painter, it is no wonder that ſome remote parts in the picture ſhould appear diſtorted; and for a very obvious reaſon, namely, becauſe it is very poſſible that the repreſentation of a right line may, upon account of its ſituation with regard to the picture, be much longer than the original line itſelf: but that a right line placed, in any poſition whatſoever, can appear longer than itſelf, is abſolutely impoſſible, and contradictory to the very nature of the ſcience of Perſpective. But to render this ſtill plainer by an example, let AB, Plate X. Figure 1. be the plane of the picture, CD an original right line, O the point of ſight; draw OC, OD, and ſuppoſe CD ſo ſituated, that the angle ODC may be a right one: then it is very evident that C*d* is the repreſentation of CD; but becauſe ~~C*d*~~ is the hypothenuſe of the right angled triangle CD*d*, it follows that C*d* muſt be longer than CD, that is, the repreſentation is longer than the original line; but notwithſtanding this, it is certain, that to the ſpectator's eye at O, the repreſentation C*d* can only appear as the tangent DC (OD being radius) of the angle DOC, formed by the rays OD, OC, paſſing from the points C, D, to the eye placed at O. For a ſecond example of this kind, let it be required to draw the repreſentation of a range of columns parallel to the picture. If they are drawn according to the ſtrict rules of Perſpective, then that column which is in the center of the picture will be the leaſt, and conſequently thoſe on each ſide of it will be larger and larger continually, the farther they are removed from the center of the picture, (if the original columns are all of the ſame diameter.) But to explain this more fully, let PP repreſent the plane of the picture, Plate X. Fig. 2. H,G, and I, the centers of the ſections of three columns, whoſe repreſentations we are to find upon the picture; and let E be the place of the ſpectator's eye: draw the tangent E*b*, E*g*, EA, EB, E*i*, and E*k*; then will their interſections with PP determine the repreſentations *cd*, *ab*, *ef*. Now it is very evident that *cd* and *ef* are much longer than *ab*. From whence we may conceive, that the farther any column is removed from the center of the picture, the longer will its repreſentation be; and we may moreover conceive, that this increaſe of the repreſentations of the magnitude of the columns, is owing to the obliquity of the chords *bg*, *ik*, with the picture; which chords will meaſure the apparent widths of the columns, if viewed from the proper point of ſight, and will appear leſs and leſs, the farther they are removed from the perpendicular E G. But if we were to draw the appearances of theſe columns upon the picture, without having regard to any particular point of ſight, but only that the eye be placed at a due diſtance from the picture; then with the perpendicular diſtance of the eye from the plane of the picture, deſcribe an arch of a circle cutting the tangents E *b*, E *g*, E *a*, &c. and the chords of the arches ſo made by theſe tangents, will be the apparent widths of the columns to be repreſented in Perſpective. Thus if $Ea = Eb$ be the radius, then will $rs$ and $tw$ repreſent the perſpective appearances of the widths of the columns I and H reſpectively.

On the foregoing principles, I have inſerted the following problems.

### PROBLEM XXI.

To find how a given right line ſhall appear of one ſame ſize to a ſpectator's eye, though it be removed nearer or farther from it.

Let AB (Fig. 1. Plate XI.) be a line given, deſcribe any circle ABEF, paſſing through the points A and B; place the eye at E or F, and draw the rays FA, FB, EA, EB. Then becauſe the angles AEB, AFB are equal, being made from the ſame ſegment of a circle (ſee Euclid, Book III. 19 Prob.) the line AB will appear equal to the eye placed any where in the arch AEFB.

### PROBLEM XXII.

Let a right line be divided in two unequal parts into, to place the eye where thoſe diviſions ſhall appear equal.

Let AB (Fig. 2. Plate XI.) be a line divided in two unequal parts in C; inſcribe any circle paſſing through A and B; divide the arch AFB in two equal parts in F; draw FC cutting the circumference in E :Then will the eye, placed in E, ſee the point C in the middle of AB; and the lines AC, CB, appear equal, being ſeen under the ſame angles.

## PROBLEM XXIII.

Suppoſe three trees planted in a ſtraight line; the diſtance between the firſt and ſecond is five times the diſtance between the ſecond and third; it is required, to find the path a ſpectator muſt conſequently move in, ſo that the middle tree appear equally diſtant from the other two?

Let A, B, and C, (Fig. 3. Plate XI.) be the trees planted in a ſtraight line, and let the diſtance BC be five times the diſtance AB; then AB may be taken equal to 1, and BC equal to 5. Produce CA to D, ſo that BD may be equal to twice the product of the two diſtances, divided by the difference of thoſe diſtances, that is, make $BD = \frac{\text{twice } AB \times BC}{BC - AB}$; which, in this particular caſe, where the diſtances are 1 and 5 becomes $2\frac{1}{2}$. Upon BD, as a diameter, deſcribe a circle: then in whatever point E of the circumference of this circle the ſpectator ſhall happen to be placed, the point B will (to him) appear to divide the line AC equally.

It will ſtand thus

$$\begin{array}{ll} & AB = 1 \\ & BC = 5 \text{ then } 1 \times 5 = \underset{2}{5} \\ BC - AB = 4 \text{ the difference} & 4\overline{)10}(2\tfrac{1}{2} = BD \text{ for the diameter of the circle.} \end{array}$$

For a demonſtration of this conſtruction, ſee Simpſon's Algebra; or Sir Iſaac Newton's Univerſal Arithemitic, page 143.

## PROBLEM XXIV.

In Fig. 6. Plate XI. B repreſents a figure or ſtatue, of a given height, placed in a wall (of a given altitude) perpendicular to the horizon, and at a given diſtance from the ground line KL. A repreſents another figure, whoſe height (greater than the former) is alſo given; placed upon the top of the wall; it is required to determine the point E, in which the ſpectator's eye being placed, the figures ſhall appear of equal altitude.

Draw an indefinite right line ALH (Fig. 4. Plate XI.) perpendicular to the horizon; let AB and CD be the given heights of the propoſed figures, reſpectively placed at their proper diſtances from each other, and from the ground line KL. Now to find the point E, proceed thus; from the middle of AB erect *f*F perpendicular to AB, equal to A*f*; from *g*, the middle of AB, erect *g*G perpendicular to CD, and equal to *g*C; through the points F and G draw a right line, cutting the indefinite one ALH in H. Upon AH, as a diameter, deſcribe a ſemicircle, and it cuts JE, drawn perpendicular to ALH, as at E, will be the required place of the ſpectator's eye, from whence the propoſed figures will appear of equal heights.

## END of the THEORY.

## OF

# PRACTICAL PERSPECTIVE.

IN this practice of perſpective, I have thought it convenient, that the vaniſhing line to the *ground* plane, ſhould be called the *horizontal line*, and the interſection of the picture with the ground plane, the *ground line*. In order to underſtand this well, obſerve the following general definitions.

Let the planes ABCI, DEO, Plate III. Fig. 1. be ſo raiſed as to ſtand parallel to each other, and bring the plane ACO parallel to the plane FGH; then

ABIC, is the picture, FGH, the ground plane.
ACO the horizontal plane, AC the horizontal line.
O the point of ſight; the line OS the diſtance.
GF an original line; *gf*, the perſpective preſentation of it; and B the interſecting point.

The point of ſight is often brought in the picture; then in that caſe, S is that point.

## CHAP. I.

### *Shewing the manner of putting any plane ſurface into perſpective.*

The diſtance the ſpectator ſhould ſtand from the picture (or what is meant by the point of diſtance) is very eſſential; for on a right choice of it depends the agreeable appearence of objects. If that point be placed very near, the repreſentation of ſome objects will be greater than their originals, (which has already been demonſtrated in the preceeding chapter); but this muſt be left to the diſcrtion of the artiſt, whome experience will improve.

I have ſet down the following method as an attempt towards finding an agreeable diſtance.

### EXAMPLE I.

*To know how far the diſtance ſhould be, ſo that the repreſentation and the appearance of an object on the picture ſhall be nearly alike.*

Deſcribe a circle with a radius equal to the ſuppoſed diſtance, and obſerve what arch of that circle may appear to the eye nearly a ſtraight line; take the chord thereof for the width of your picture, for as much as the picture exceeds the width, ſo much will the object appear different from its repreſentation on the picture, as has already been ſhewn. For lines may be drawn from the object, ſo placed to the ſpectator's eye, as to form an angle too large for the eye to take in.

For let the line ED (Fig. 1. Plate XII.) repreſent the ſection of a picture, OC the diſtance; deſcribe the arch *edf*. If the width of the picture is made equal to AB, the repreſentation of an object on that picture will be as near to its appearance as the line AB is to the arch *a*C*b*; the more you increaſe your picture towards F or E, the greater the difference will alſo increaſe, it being always as a right line to the circumference of that arch, as may be ſeen by DF and *df*. The diſtance here OC is 5, and the ſize for the picture is 1; and AB is the ſide of a ſquare picture.

But as a picture in this proportion would not do for the plates in this book, as the point of diſtance would extend too far, I have taken ſuch diſtances as would admit of thoſe points.

This you may look upon as a general maxim, viz. that if the picture be extended from E to F, that the diſtance between the point of ſight and the point of diſtance muſt be at leaſt equal to OC; the angle EOF forming an angle of about 50 degrees; which will do very well when there is no object too near the ground line GL.

On the horizontal line, on each ſide of the center or point of ſight C, ſett off the diſtance OC; but when the plane of the picture is not wide enough to receive the two points of diſtance, you muſt make only one; and inſtead of placing the point C in the midle of the picture, it may be placed on one ſide.

## EXAMPLE II.

*To find the repreſentation of a point in the picture.*

Let HL (Fig. 2. Plate XII.) be the *horizontal line*, C the *center*, O the point of *ſight*, DB the ground line, and A the given point. Through A draw any right line AB, meeting the ground line in B. Make OH parallel to AB, interſecting the *horizontal* line in H; for the vaniſhing point of the line AB, draw BH; and from A draw the ray AO, interſecting HB at *a* for the point required.

## EXAMPLE III.

*To find the repreſentation of a given line*

Let HL (Fig. 3. Plate XII.) be the horizontal line, continue the original line AB to the ground line D*d*, in *d*; the vaniſhing point H is found as in the foregoing example. Draw *d*H, the rays BO, AO, cutting the line *d*H in *b* and *a*; give *ab* for the repreſentation of the line AB.

## EXAMPLE IV.

*To find the repreſentation of a line when the original line paſſes through the center of the picture.*

Let HL (Fig. 1. Plate XIII.) be the horizontal line, GF the ground line, C the center, OC the diſtance, and DB the original line. Continue DB to the ground line at *e*, and draw at pleaſure two lines B2, B1, parallel to each other, cutting GF at 1 and 2: make OE parallel to either of them: draw 2E, 1E; then draw *e*C, interſecting them at *b* and *b*; join *db* for the repreſentation of the line DB. For the projection of the line AK, which is perpendicular to the picture, its vaniſhing point is C; draw KC and OK, and *ka* is the projection of AK: C is the vaniſhing point of the lines AK, and BD.

## EXAMPLE V.

### *To find the repreſentation of a given triangle.*

Let ABD (Fig 2. Plate XIII.) be the original triangle. HL the horizontal line, GF the ground line, and OC the diſtance; produce the ſides AB and DB, to the ground line at 1 and 2. Make OH parallel to AB, and OL parallel to DB, cutting the horizontal line at H and L the vaniſhing points of the lines AB and DB; draw 1L, 2H, interſecting each other in *b*; draw the rays AO, DO, cutting the lines 1L, 2H, in *a* and *d*; join *abd* for the repreſentation of ABD.

Note. As AD is parallel to the ground line, the finding either point *a* or *d* (in the picture) will do; the line *ad* may be found by drawing a line through the one that is found parallel to the ground or horizontal line.

## EXAMPLE VI.

### *Having an original ſquare given, to find its perſpective repreſentation.*

Let LH (Fig. 3. Plate XIII.) be the *horizontal* line, GF the ground line, O the point of ſight, ABDE the original ſquare, the ſides AD, BE, being perpendicular to GF, make OC perpendicular to LH, cutting the horizontal line at C, for the vaniſhing point of the ſides AD, BE, and center of the picture; continue AD, BE, till they interſect GF at 1 and 2; draw 1C, 2C, then produce the diagonal DB to the ground line at 3, and make OH parallel to DB, cutting HL at H, then H is the vaniſhing point of DB; draw 3H, cutting 1C, 2C, at *b*, and *d*; make *ba*, *de*, parallel to GF (their originals being ſo), cutting 1C, 2C, in the points *a*, *e*; join the points *abde*, for the Perſpective Repreſentatioon of ABDE.

When CH and CL on the horizontal line are made equal to the diſtance of the picture (as in Fig. 1. Plate V. Problem 2.) then H and L are the vaniſhing points of the diagonals of a ſquare that has two ſides parallel to the picture, as H is the vaniſhing point of BD; (for the angle COH is of 45 degrees, ſo is the angle ABD;) then OH is parallel to BD, and C (the center) is the vaniſhing point of the ſides AD, BE. In moſt of the following examples, the point of diſtance is laid on the horizontal line, as being moſt convenient for the practice.

### EXAMPLE VII.

*To find the repreſentation of a ſquare lying obliquely to the picture.*

Every thing being prepared as in the foregoing ſcheme, continue the ſides BA, and ED (Fig. 4. Plate XIII.) of the original ſquare (to the ground line GF) in the points 1, 2, 3, and 4; make OL parallel to AD, BE and OH parallel to BA and ED; draw the lines 4 H, 3 H, 2 L, 1 L, interſecting each other at the points *a, b, e d,* the perſpective of ABDE.

### EXAMPLE VIII.

*To determine the repreſentation of a pentagon, from an original given.*

Let ABCDE, (Fig. 1. Plate XIV.) be the original figure. HL is the horizontal line, O the point of ſight, GF the ground line. N, P, Q, M, the interſecting points on the ground line, of the ſides AB, CB, DC, EA; L is the vaniſhing point of the ſide CB, H that of the ſide AB, K of the ſide EA, and I of the ſide DC (all found by drawing lines through the point of ſight parallel to thoſe ſides.) Draw PH, QL, interſecting each other at *b*; and the lines MK, IN, to interſect QL, PH, in *c* and *a*; draw the rays DO, EO, interſecting NI, MK, in *d* and *e*; join thoſe points, you have the projection required. Polygonial figures of any number of ſides are put in perſpective in the ſame manner.

## EXAMPLE IX.

*To find the perſpective of a circle or any given curved line.*

Let K (Fig. 2. Plate XIV.) be the horizontal line, EF the ground line, O the point of ſight, and ABC the original circle given. Draw at pleaſure from any point B, the tangent line BF, interſecting the ground line at F, and as many lines through the circle (all parallel to BF) as you pleaſe; make OK parallel to BF, then is K the vaniſhing point common to them all. The point *b* of the original point B is found thus; draw FK, and from B draw the ray BO, interſecting FK at *b*, the point ſought. The point *a* is found by drawing EK, and AO, cutting EK at the point *a*. The points 1, 2, 3, *c*, *d*, and others are found in the ſame manner. Theſe points muſt be joined carefully for the perſpective repreſentation.

## EXAMPLE X.

*To find the projection of a circle by the means of a ſquare.*

Let 3 4 5 6, (Fig. 3. Plate XIV.) be the given circle, make the ſquare ABCD about it, touching the circle at the points 8, 9, 7, 10; if the ſides AD, BC be perpendicular to the picture, the center I will be their vaniſhing point; draw the diagonals BD, AC, cutting the circle in the points 3, 4, 5, 6; draw lines through 4 6 and 3 5, meeting the ground line FG: then through the interſecting points 2, 1, &c. draw lines to the point I; produce the diagonal AC, to F, and make KH parallel to FC, cutting the horizontal line HL, in H; draw FH, which will give the points *a*, 4, *e*, 5, *c*, the ſides AB, DC, being parallel to the picture, draw *c* 10 *d*, 5 6, 8 *e* 7, 4 3, *a* 9 *b*, all parallel to the ground line, join 9, 4, 8, 10, 7, and 3 for the repreſentation of the circle.

### EXAMPLE XI.

*To find the repreſentation of a circle on any part of the picture, by having the diameter given.*

Prepare the picture in this manner: Let AB (Fig. 2. Plate XV.) be the horizontal line, OC the diſtance, and brought in the picture on the line AB, in A and B; C the center. Set one foot of the compaſſes in the point B, with the opening BO deſcribe the arch OL; and with AO, the arch OV; let 3 7 (parallel to the horizontal line) be a diameter in the picture, through the middle in 9 draw C 5 1, (which will repreſent a line perpendicular to it); from A and B, draw lines through the point 9. To find the points 1, 2, 4, 5, 6, and 8; from B through the point 7, draw a line, interſecting the line 5 9 1, at 1, a line from 3 to B gives the point 5 on C 1; from V and through 7, a line being drawn it will cut the line A 9 8 at the point 8; and a line from 3 to V gives the point 4, on the line A 9; from L through 3, a line being drawn cuts B 9 at the point 2; and another from 7 through L, gives the point 6. Then will the lines 91, 92, 93, 94, 95, 96, 97, 98, repreſent equal lines. This method may ſerve for an octogon, a ſquare, and is the readieſt way to repreſent circles within one another.

## CHAP. II.

*Practical Perſpective of ſolid Bodies.*

### EXAMPLE XII.

*To find the projection of a cube from an original given, having one ſide of it parallel to the picture.*

Let HL (Fig. 1. Plate XVI.) be the horizontal line, OC the diſtance, FG the ground line, and ABCD the plan (or orthography) of the original cube; find the perſpective ſquare *abcd*, of

ABCD, (by Example VI.) At the point 2 on the ground line, raiſe a perpendicular 2 *e* equal to the height given; on the points *a*, *b*, *c*, *d*, raiſe perpendiculars at pleaſure; from the point *e* draw the line *e* C, cutting the perpendicular *b d*, at 6 and 7; from the points 6 and 7 draw lines parallel to *a b*, cutting the perpendiculars *a* and *c* at 5 and 4; join thoſe points, as you ſee in the ſcheme, for the ſolid figure.

N. B. The line E 2 of the height given, may be put any where on the ground line, as at F; and draw the lines F N, K N, to any point in the horizontal line, as at N; then to find the height the perpendicular *a* 5 is to be (on the picture), draw *e f* parallel to the ground line, meeting F N at *f*; and draw *f h*, perpendicular to G F or H L for the height of *a* 5; the perpendicular *c* 4, is equal to *g l*; and *d* 7 (being on the ſame parallel *c* 4) is of the ſame height.

Obſerve, that all lines that are parallel to one another in the original, vaniſhes in the picture, to the ſame vaniſhing point as you ſee 54, 67, vaniſhes at C the vaniſhing point of *ac db*, which is the reaſon of putting the line *e* 2 on one of the interſecting points.

## EXAMPLE XIII.

*To find the repreſentation of a pyramid, the ichnography (or ground plan) being given, whoſe poſition is oblique to the picture.*

Let HL (Fig. 2. Plate XVI.) be the horizontal line, OS the the diſtance of the picture, L the *vaniſhing point* of the ſides CD, EF, H the vaniſhing point of CE and FD, the line AB on the *ground* line is of a given height. Find the *Perſpective* plan *c d e f* (by Ex. VII.) draw *ed*, *ef*, to find the center *a* of the Perſpective plan; on *a* raiſe a perpendicular at pleaſure; from L through the center *a*, draw a line to the ground line, at that interſection ſet the given line AB; draw BL cutting the perpendicular *ab*, at the point *b*; join *bc*, *bd*, *be*, *bf*, and complete your pyramid.

## EXAMPLE XIV.

*To put a cube in perſpective ſtanding obliquely to the picture.*

Every thing remaining as in the foregoing example, find the Perſpective ground plan *a b c d,* of the original ABCD, (Fig. 3. Plate XVI. by Ex. VII.) at the point 1 ſet a line 1 F for the given height, on the points *a, b, c, d,* raiſe perpendiculars at pleaſure; draw FL, cutting the perpendiculars *a, d,* at 5 and 6; from H, through 5 and 6, draw lines, cutting the perpendiculars, *b, c,* at the points 8 and 7, then is you ſolid figure finiſhed.

## EXAMPLE XV.

*To put a chair in perſpective from a given original plan, whoſe poſition is parallel to the picture.*

Let ML (Fig. 1. Plate XVII.) be the *horizontal* line, N the *center* and vaniſhing point of the ſides AC and BD, of the original plan X. M, the vaniſhing point of the diagonal DA; the ſmall ſquares on the original plan are the plans of the legs; find the Perſpective plan *a b c d,* (by Ex. VI.); from the points 2, 3, on the ground line 1 J, draw 3 N, 2 N, cutting KM and there determine the little ſquare, whoſe parallel ſide is 5 *a*; continue a line to 6 it will give *b* 6, and ſo finiſh the other ſquares; at *d* and *c,* and on each point *a,* 5, 6, *&c.* of the Perſpective plan, raiſe perpendiculars at pleaſure. At the point 4 on the ground line 1 J, ſet the perpendicular RS, with the heights of every part of the chair, marked *z, y, x, g,* 7, 8, and from thoſe points draw lines to the point N, cutting the line *l b,* in *l, u, v, t, e, f, h*; from *f* draw *f* UT, cutting the perpendicular *a* at T, draw TN cutting the line *c k* at *k*; draw *hk,* parallel to UT, then will three ſides of the chair be completed; a line from *f* to N finiſhes the back. At the point U draw a line to the point N to croſs the uprights, in thoſe little lines U and *k*. To find the rail *s*, draw 7 N, cutting *lb* at *b*; and from *b*, draw a ſmall line acroſs the perpendicular on the point 6, parallel to *b a*; and from that interſection draw a line through N, to the leg *d,* which will give the upper ſide of the rail *s*;

the ſame muſt be done for the other ſide from the line under *b*: it is eaſy to imagine the rails Q, and P, only they don't go acroſs the uprights but half way, as the line RS, on the point 4 is in the ſame plane with the back of the chair. It finiſhes only that ſide: the other ſides are found in the ſame manner as the cube.

## Example XVI.

*To give the repreſentation of a ſquare table.*

Every thing remaining as in the foregoing example, let ABCD, (in fig. 2.) be the ground plan of the table, and the little ſquares thoſe of the legs; 1, 7, 6, 2, their interſecting points on the ground line. The line on the point 2, the *height* of the table with the parts, marked 5, 4, 3: *a*, *b*, *c*, *d*, the *Perſpective* plan of ABCD; *a q*, *p c*, the thickneſs of the feet; the rail *f* is caried parallel to FJ, cutting the perpendicular *q*H, through that interſection and *f*; draw lines to the point N interſecting the back feet at *h*, and from thence a line parallel to *a c*, through the leg *b* G; The lines that ſerve to ſhew the thickneſs of the legs are contrived in the ſame manner; GHEF, repreſents the top. The ſcheme is ſufficient without any farther explanation.

## Example XVII.

*To find the repreſentation of a cylinder ſtanding perpendicular on the ground, by having a line given in the picture as the diameter of its baſe.*

OH (Fig. 1. Plate XV.) is the horizontal line, O H is equal to the diſtance of the picture but brought in the picture, O the center (or point of ſight brought in the picture) and alſo the vaniſhing point of all lines perpendicular to the picture, and GB the ground line. This manner of preparing the picture is moſt convenient for the practice of Perſpective, and obſerve that the points of diſtance, are the vaniſhing points of the diagonals of a ſquare if one of the ſides be parallel to the picture or ground line, the point V in this example is found by drawing a line through O perpendicular to OH, and equal to OH, and find V as in Ex. XI. Then let *a b*, be the line given; parallel to OH biſect it in *x*, and draw O*x*B, O*a*E and O*b*F; then from the point H draw H*x*F cutting OF in F, and O*a* in G; through F and G draw GI and FE parallel to the horizontal line; then join EL and FG for the diagonals of the ſquare; then draw V*a*, *b*V, cutting EI in 5, and 7; as you have but one point V, draw O7, O5, which will cut FG in 8 and 6; join the points B, 6, *b*, 7, *c*, 8, *a*, and 5 for the circle in the picture; on the points *a*, 5, B, 6 7, *c* 8, *x*, raiſe perpendiculars at pleaſure, and let BD be the height given; draw DO cutting the lines *xe*, *cf* at *e* and *f*; through *e*, draw a line parallel to the horizontal line, cutting the lines *bb*, *ag*, at *b* and *g*; from V through *g* draw a line cutting the perpendicular 5 1, at 1, and a line from *b* to V, gives the point 3; on the line 7 3, from 3 draw a line parallel to *gb*, it will give the point 4 on the perpendicular 84, and through 1 draw a line parallel to *gb*, it gives the point 2 on the line 62, then join *b*2D1*gf*34 for the circle at top, and if the two circles be joined by a number of right lines, it will form the cylinder required. But as in all round bodies we cannot ſee quite half though ever ſo remote from it. Then to find on the picture as much as we can ſee, at any diſtance: when the circle *a*5B6*b*7*c*8 is found, take the longeſt line from *x* to the circumference in the arch *bca* as *ix*, and *xs* in the arch *a* B*b*, and on *i* and *s* erect the perpendiculars, *ir*, and *sw*, cutting the circle at top at *r* and *w*, then *ir* and *sw* will be the out lines of the cylinder, being as much as can be ſeen from that point of ſight.

## EXAMPLE XVIII.

*To ſhew the perſpective of a cylindrical body lying on the ground at right angles with the picture.*

The picture having every thing the ſame as in the foregoing example, the line *a*C (fig. 3.) vaniſhing at O, is the axis running through the cylinder. All the circles on the body are parallel to the *picture*, and are deſcribed with compaſſes; draw B*b*, to the point O. On the *ground* line BG, ſet the meaſures for the diſtances between the circles, and length of the body. To find the circle *c d e* on the line BG, let the point A be the diſtance for one circle: draw AH, which gives the point *e* on B*b*; on that point raiſe a perpendicular through the line *a*C, and *a* is the center for that circle. If the object ſhould repreſent a cannon, as they diminiſh at the muzzle, let C be the ſmall end, draw a perpendicular through C, cutting B*b* in *b*: let 3 4, on the line *ec*, be equal to the ſmall end wanted; draw from 3 and 4 lines to O, cutting the line C*b*, in 1 and 2; ſet one foot of the compaſſes in C as center, with the opening C 1, or C 2, deſcribe a circle; divide your circles into as many equal parts as you pleaſe, and join thoſe diviſions together with ſtraight lines, as *c* 2, *d*R, *&c.*

## EXAMPLE XIX.

### *To put any kind of wheels in perſpective.*

This example is the repreſentation of a water-wheel that ſtands perpendicular on the ground with coggs. HL (Plate XVIII. Fig. 1.) is the *horizontal* line, H the point of ſight (in the picture), E the point of diſtance laid on the horizontal line, CM the *ground line*, VHV the *vaniſhing line* of the plane BEGI; EF or 5 D the thickneſs; from the point 5 draw a line 5 G to the *center* H, and from C draw CL to terminate the line 5 G at G; a line drawn from K (the center of the original circle) to the point L, gives *e*, on BG; to finiſh the perſpective ſquare BGIE (having found the line BG) draw 5 E, GI parallel to the line VHV, on the line VHV, which is perpendicular to the line HL; on each ſide of H ſet the diſtance HL as points of diſtance on that line; then ſetting one foot of the compaſſes in one of thoſe points, and the other in L, deſcribe the arches LV, and the point VV will ſerve to determine the points on the diagonals to form the circle (for the better underſtanding this, I have made the ſcheme, Fig. 2. only ſmaller); and from the two points of diſtance, draw lines paſſing through G and 5, interſecting each other in O, cutting GI in I, and 5 E in E; then is GIE 5 the ſquare, GE, 5 I, the diagonals, O the center of it,

and

and draw HO*a*, and *e*O*f* for the two diameters; after that find a circle (by Ex. XI. or XVII). Then on the point D for the thickneſs, draw DF, parallel to 5E, and with that line finiſh another ſquare as is repreſented by the dotted line, and draw from each corner diagonals, as in the firſt ſquare: when your firſt circle *edt* is finiſhed, ſet off any given height for the coggs, on 5E as 6, draw 6H, cutting *ef* in the point to form the circle of the wheel, and ſo on for the next. To divide the wheel, draw a quarter of a circle as 1 2 3 4, and divide that in as many parts as you pleaſe, it is here in 4; from each of theſe points draw perpendiculars to the ground line, and from their interſection with it draw lines to the point of diſtance L, cutting BG; then draw perpendiculars through one half of the perſpective circle, and from thoſe points draw lines through the center of the wheel acroſs, to find the others. For example, to find the points *b* and *f*, from B draw a perpendicular, cutting the circle in *b* and *f*; from *f*, thro' the point O, draw a line croſs the circle for the point *k*. For the thickneſs of the coggs, (ſuppoſing them in the original to diminiſh to the center) from the the point *n* of the cogg *f*, draw a line thro' the point O acroſs the circle, gives the point *l*. Thoſe lines being drawn to the line VHV, will give V, T, for vaniſhing points of the ſides *f*, *n*, by this means every circle is divided; from every point *f*, *n*, draw lines parallel to the horizon, interſecting the outward circle at *p*; and from the vaniſhing point V of *n*, draw *p*V; from *d* draw a line parallel to *n*, *p*, cutting *p*V at *c*, then is *np*, *dc*, a complete ſide of one of the coggs. To find them without uſing vaniſhing points, as it is often very troubleſome to get, make the circle *c*, then from *d* the parallel line cutting that circle in *e*, join *cp*, the eight ſpokes are the principal lines made uſe here for the circle, and as it is ſuppoſed muſt paſs through the middle of the thickneſs of the wheel, proceed thus, divide 5D, in P, and a line drawn from P, to the point of diſtance on the line VHV, will give the line through the middle of the ſpoke; but to do it without that point of diſtance, draw the line *rms* parallel to the *horizontal* line, divide that in two equal parts at *m*, draw *m*P, proceed for every other in this manner, join the extremity of thoſe croſs lines in a circle, to ſhew the inſide thickneſs of the wheel.

## EXAMPLE XX.

*How to put doors in perſpective, and to deſcribe pannels thereon.*

Let Fig. 1. Plate XIX. repreſent the inſide of a room, HL *the horizontal* line, C the *center*, OC the *diſtance*, V a point to find the ſemicircle on the floor, (deſcribed by the door opening) UR a line equal to the breadth of the door, continue U to the *center* C, draw a line from the point R to the point of diſtance (removed on the horizontal line, but omitted on the plate for want of room) interſecting UC at X; let UII, be a given height, draw IIC, and on X raiſe a perpendicular cutting IIC, at III, then is IIIX, IIU, the aperture of the door; draw RC, and from X a line parallel to UR, cutting RC at T, draw XR for a diagonal, X the center, from V through T draw a line cutting XR at Y, join UYT for a quarter of a circle. The other quarter, if need be, is found, by finiſhing the ſquare XT*b*Z, and draw the diagonal XZ, and from C and Y draw a line cutting XZ at *y*; join T*yb* for the other quarter; chuſe any point on the ſemicircle for the opening, as F; draw a line from that point through X, to the horizontal line at H for the vaniſhing point of lines parallel to that, and from H, through III draw a line at pleaſure; on F raiſe a perpendicular cutting III*a* at *a*, then III *a*, XF repreſents a door opened. To form the pannels on the door, draw FE parallel to the horizon; ſet the foot of the compaſſes in H, and with the opening HO draw the arch OL on the horizontal line, and L is the point of *diſtance*, and H the *vaniſhing point* of the door; for the point L, through X, draw a line, cutting FE at E, then EF is equal to the original of FX, ſet on FE, the meaſures 1, 2, for the width of the pannels, and from thoſe points draw lines to the point L, cutting XF at 3, and 4, from any points on F*a*. For the height, draw lines to their vaniſhing point H, cutting the perpendicular raiſed on 3, and 4, which terminate the pannels A and B.

The door, Fig. 2. at the end of the room is found in the ſame manner; the line KN the breadth, and KMPN a ſquare made of that line, K is the center or hinge of the door, with the point V on the horizon; find the point *p*, and finiſh the quarter of the circle, KW height of the door, the dotted lines from K and W vaniſhes to the ſame point on the horizon.

Fig. 3. is the repreſentation of a trap-door on the floor: G 9 the breadth, and G D 7 9, a ſquare in perſpective of the aperture, the door, D Q 6 7, is found the ſame as Fig. 1. or Fig. 2. KIDG is a dotted ſquare, equal to the ſide DG of the aperture, *l* O the vaniſhing line for it, C the center, OC the diſtance, and the points J and L, to find the circle, then proceed as in the other Example.

## EXAMPLE XXI.

*How to give the perſpective repreſentation of rooms, doors,* &c. *The point of diſtance is ſet on the horizontal line in this and the following examples.*

Let HL (Fig. 1. Plate XX.) be the *horizontal* line, S the *center*, L and H points of *diſtance*, AU the breadth and depth of the room (ſet off on the ground line), AW the height. Draw AS, US, then draw AL interſecting US in C, draw CB parallel to AU, then will ABCU repreſent the floor in perſpective. To finiſh the ſides, raiſe perpendiculars on A, B, U, C, draw WS, cutting BD in D; then as AU is parallel to the picture, draw DZ parallel to HL, cutting CZ in Z.

To ſet a door on the ſide of a room. On the ground line AU, let *a*U be the width of the door; and U *b* the depth it muſt be in the room, draw *b*L cutting US at F; from F draw EF at pleaſure parallel to AU, draw *a*S cutting FE at E; and the line EL will cut US at *e*; on *e* and F raiſe perpendiculars to make the door, the ſemicircle *e*EG and door is found as in the foregoing example; K is the vaniſhing point of the door I, H the vaniſhing point of the door N, *t* of the door T, and I of the door M. The projection of the chimney-piece is found as the door, by ſetting on the ground line the meaſures, and proceed as with the door. Fig. 1, 2, repreſents the back part of an upper room, *r* another behind it, and T the door opening within it; E another room under the room AWBDZCU, and R a room and pavement behind it; V the vaniſhing point of the ſtairs.

## EXAMPLE XXII.

### *How to put windows in a room from any given Meaſurement.*

Let HL (Fig. 2. Plate XX.) be the *horizontal* line, A*m* the *ground* line, L point of *diſtance*, H the point of ſight, the points *m*, *n*, *p*, *q* the breadth of the panes and frame, and B, C, D, the height of them; lines from *m*, *n*, &c. to the point L, gives on AH the points 1, 2, 3, 4; on theſe points raiſe perpendiculars; the perpendiculars 1, 4, gives *a*W on the line QH, for the ſquare of the window-frame. To give the thickneſs, and repreſent the ſides, take OQ for the thickneſs of the wall, draw OH, from *a* draw a line parallel to the horizontal line, interſecting OH: then *kk* is the upper thickneſs; and where the line *a* interſects OH, draw a line parallel to it; *a* 1 gives you the ſide: where the perpendiculars 1, 2, 3, interſect the under frame of the window, draw parallels to HL acroſs the window-ſeat, interſecting PH; at thoſe points raiſe perpendiculars for the uprights of the ſaſhes, and from B, C, D draw lines to the point H to finiſh the ſquares, continue *a*WQ to interſect the line *b* 5 (at the end of the room); at 5 draw 5*l* parallel to HL, for the top of the window; do the ſame to get the bottom, as is ſhewn by the dotted lines 1*t*; draw *xt* parallel to *lf*; and from *m*, *n*, *p*, *q*, draw lines to H, cutting *bb* at *e*, *d*, *c*. To find the thickneſs *gf*, continue O*k*, cutting the line 5*b* at 6, and from that point draw a line parallel to *lf*; draw *fg* to the center H, interſecting 6*g* at *g*; from that point draw a line parallel to *fe*, for one ſide of the window, and draw *z*H, interſecting that line; from that interſection draw a line parallel to *xz*, for the thickneſs of the bottom. The croſs barrs are found by drawing lines from the points 7, 8, 9, to W, and from their interſections on the line *g*, draw lines parallel to *xz*.

EXAMPLE

## EXAMPLE XXIII.

### *To put a Tuſcan pedeſtal in perſpective.*

On the *ground* line AK (Fig. 2. Plate XXI.) raiſe a perpendicular *a c*, for the height of the members, 1, 2, 3, 4 *b c*, for the projection of thoſe members take 4*e*, 3*d*, *&c.* On any point of the ground line raiſe the perpendicular AC; on it ſet the points 1, 2, 3, 4, B, C, equal to thoſe in *a c*, as A*i* the height of the plinth, *&c.* At 1 draw GF parallel to AK, 1G or 1F is equal to half the width of the plinth; from G and F draw lines to the center H. On the ground line ſet the depth (equal to the line GF); from G to K; draw K to the *point of diſtance* I, cutting GH at the point *l*, raiſe a perpendicular, and finiſh the plinth; from each corner draw the diagonals F*b*, GL, interſecting each other in O, the center of that ſquare; ſet the different projections from the line *a* to *d*, and put them on from 1 towards D, draw 1H, and from the firſt point on 1D draw a line to I interſecting 1H, at the interſecting point draw a line parallel to GF, cutting the diagonal GO, and from thoſe interſections draw lines to the point H, interſecting the diagonal at *b*; on each point raiſe perpendiculars for the height of the fillet, then from the point 2, the height, draw a line 2H, cutting the perpendicular on the line 1H, for its proper height; from the interſection draw a line parallel to GF, interſecting that perpendicular, on the diagonal GF, and finiſh the fillet in the ſame manner as theplinth; the other members are laid on in the ſame manner. For the capital of the pedeſtal, proceed as for the baſe. As the flatb and BC projects fluſh with the plinth, from B to E lay the profile of the members, draw BH, interſect it by EI, and finiſh every member before you lay another on it.

## EXAMPLE XXIV.

*How to lay members of architecture on one another.*

Let ABCD (Fig. 2. Plate XXII.) be a ſquare plan in Perſpective, where on it another is to be put on equally every way, GP is the *horizontal* line, G the *center*, P the *diſtance*, *rx* is the *diſtance* it is to be laid on; draw the diagonals AC, BD; where *r*P interſects *x*G, draw the line *ad* parallel to AD, cutting the diagonals at *a* and *d*; from thoſe points draw lines to G, cutting the diagonals B and C at the points *b*, *c*; then will the plan *a b c d* be placed equally on ABCD.

## EXAMPLE XXV.

*In this example is the perſpective of the Tuſcan entablature.*

Let H (Fig. 1. Plate XXI.) repreſent an original ſection, on the line AD are ſet the height of the parts, *viz.* 1, 2, 3, 4, *&c.* and from thoſe as AB, 1 *b*, 2 *c*, 3 *e*, *&c.* are the diſtances they are from that line, (this figure is put on the plate upſide down for want of room) LH the *horizontal* line, H the point of ſight, and L the point of *diſtance*, laid on the horizontal line. Let FE be equal to the whole width of that member, divide it into two equal parts at *a*, let fall the perpendicular *ad*; on that line ſet the meaſures from AD, 1, 2, 3, *&c.* with the line FE make a perſpective ſquare plan, draw diagonals acroſs, from *a* draw a line to H, from *b* draw *b*L cutting *a*H at *n*, through *n* draw a line parallel to FE cutting the *diagonals* at M, M, draw MH till it cuts the other diagonal; on *n*, M, M, raiſe perpendiculars. From the point 1 on *ad* draw a line to H cutting the perpendicular *n*, and from that point draw a line parallel to FE, cutting the perpendiculars M, M, and finiſh that member as if it was a plinth; then on the corners at F and E draw the quarter round, and proceed the ſame for the reſt, obſerving that every member lies properly on the diagonal of the preceding.

## EXAMPLE XXVI.

*In this example is the perſpective of the Tuſcan baſe.*

Let X in Plate XXII. Fig. I. be an original ſection made as in Plate XXI. Fig. 1. But to give the proper repreſentation of the ſwell of the torus, I have divided the original in two equal parts, horizontally as marked 2, and vertically marked *x*; and where the line *x* cuts the torus, lines are drawn parallel to *x a* againſt *a* 5, in the points 1 and 3, to expreſs the ſwell. The baſs is ſuppoſed parallel with the picture, and the plinth BD, whoſe height is AP, is put in Perſpective in the ſame manner as a ſquare; SL is the *horizontal* line, S the point of ſight, and L the *diſtance*; on the ſurface of the plinth draw diagonals. Their interſection *o* is the center of the baſe, through that point *o* draw a line parallel to DB, for a diameter, and from the S through *o* draw a line for the other, and give the point A on BD, with the point V make the circles on the plinth, as has been ſhewn in Plate XV. Fig. 2. On A 5 ſet the height of every member, as 1, 2, 3, &c. on A J are the meaſures for their projections. The point *c*, for one perſpective circle, is found by the line JL, cutting AS in that point; the other points on that line are found in the ſame manner: thoſe points are afterwards carried on one of the ſemidiameters, by continuing a line from thoſe points to the point of diſtance interſecting the ſemidiameter, then with a pair of compaſſes carry them over to the other, as *a*, *b*, *c*, then from V and thoſe points finiſh your circles, on the center *o* raiſe the perpendicular K*o*, and through every point 1, 2, 3, &c. on the line A 5, draw lines to the point S, cutting K*o* in 1, 2, 3, &c. On that part of the circle you intend for a ſection, draw a line through the line K*o*, to the horizontal line, and the interſection will be the vaniſhing point; from that point draw lines through each point on K*o*, cutting the perpendiculars on thoſe points for the circle of that ſection. For Example: Suppoſe for the ſection on the line AH, S is the *vaniſhing point* for it, on the point, *a*, *b*, *c*, on AS, raiſe perpendculars, and from S through every diviſion on K*o* draw lines, cutting them. The firſt point on A marked 2, for the middle or greateſt ſwell of the torus; on the ſecond, in two places to expreſs the turning of the round; on the third, at *b* 4 and 5, the point *b* joining the plinth, and 4 where the fillet begins, 4 and 5 the thickneſs of the fillet, on *c* at *d* the ſhaft begins; do the ſame on as many parts of the circle as you think neceſſary to complete your figure. Then join carefully the points 4, 4, 3, 3, 2, 2, &c. in curves. R is part of a ſection behind that on the diagonal B, E is part of another; to finiſh this figure to a great exactneſs, girt the ſections with as many of thoſe curves as will fill the ſpaces.

When the learner has accuſtomed himſelf in viewing a true repreſentation, he may with only the circles complete the torus.

## EXAMPLE XXVII.

*In this example is the perſpective of the Tuſcan capital.*

On the line AJD, (Fig. 3. Plate XXII.) ſett off the meaſures as on the *ground* line, HL is the *horizontal* line, L the *center* of the picture, H the point of *diſtance* laid on the horizontal line, and V one of the points for finding the projection of the circle, CD the width of the abacus, J the middle of it, *a*, *b*, *d*, *g*, the ſeveral projections laid on from J towards D, J*r* a line for the height of the parts, as 1, 2, 3, &c. KO the center of the column, ABDC an original ſection; proceed with this as with the foregoing example.

## EXAMPLE XXVIII.

*To find the perſpective of a double croſs.*

Let LH (Fig. 3. Plate XXIII.) be the *horizontal* line, AD the *ground* line, L the *center*, H the point of *diſtance*, ABCD the extent of the croſs bar, BC its thickneſs; draw AL, BL, CL, DL, and from A draw AH, gives the points *b*, *c*, E; through theſe points draw lines parallel to AD, for *ag* on AL, and *d* on DL; on the point B ſet the perpendicular BG, with the height

of

of the parts required; then on the points *b, c, e, f,* in the plan of the upright poſt, raiſe perpendiculars at pleaſure, and through the points on BG, draw GL, KL, IL, cutting *bo* at *qr*; draw through the point *r* a line parallel to the ground line, croſs the perpendicular lines *cb,* as *mrst,* and another parallel to it through *q.* IK, K*n*, is the end of one of the croſs bars, draw KL and IL: Perpendiculars on *agd*F will determinate the points *m*, Q, *t*, &c. where the line IQ interſects *fp*, draw a line parallel to *mt*; from *tm* draw lines to the point L; a perpendicular on *g* will finiſh the ſide *m*L.

Plate XXIII. Fig. 2. is the repreſentation of a bureau with the flap open: *bik* is the circle deſcribed by the flap opening, PL the *horizontal* line, L the *center*, OOX the *vaniſhing* line of the plane BA*i*, O, O two points to draw the circle by; X the point of diſtance on OLX, A*k*MN the flap, AN the joint it turns on; proceed in this as with the door, &c. Plate XIX.

## EXAMPLE XXIX.

*To find the middle or center of any arch or building*, &c.

Let HV (Fig 1. Plate XXIII.) be the *horizontal* line, V the *center*, H the point of *diſtance*, EC one ſide of the building; through E draw HA, which produce to the ground line AG; divide AC in two equal parts in F, draw FH, cutting CE in D (the middle of EC); on D raiſe the perpendicular DL, cutting RK in L; draw a line parallel to AG, interſecting MV in T; divide that line in the middle, and deſcribe the ſemicircle LQT; then divide the arch M*t*R in two equal parts in *t*, and draw V*t* cutting the arch TQL in Q, which point is the center of the arch MNRK, which was required.

## EXAMPLE XXX.

In this example the eye is ſo placed in the air as to look down into the building, VUB and RUS (Plate XXIV.) are two *vaniſhing* lines at right angles, U is the *center* of the picture, R and S two points of *diſtance*; the line GI parallel to RS, for convenience, muſt be conceived as a *ground* line, ABCD an original arch given, PG is equal to the line BC, PA the thickneſs of the pilaſter, PI is half the breadth, and NO the extent of the capital and baſe; interſect PU by GR, to get the point *g*; LM drawn to R, gives on PU, *h*, *l*, *m*, the ſame parts of the original, as by their capitals; through them, lines are drawn parallel to GI, they terminate the ſide *aa* GI. NU gives *n*, *n*, for the projection of the baſe and capital. To turn the perſpective arch, W on the line GI repreſents its center; draw WU cutting *tt* at *x*; from R and S, through *x*, draw lines at pleaſure, for the diagonals *xv*, *vy*. To turn the arch, draw Z*tv*, Z*ty*, S*tp*; join *ty* *pvt*. The ſides of theſe pilaſters are parallel to VUB; the window 1 2 3 4 is found by drawing 2U, 5U; for the thickneſs, draw 1U, and draw 4 3 parallel to RS, cutting 1U at 3; and from 3 draw a line parallel to VB, finiſhing the bottom of the window. In this ſcheme the ground plane is parallel to the picture, therefore the ſquare pavement appears in the ſame ſquares as the original.

## CHAP. III.

### *Of inclined planes and objects.*

Fig. 1. Plate XXV. is the perſpective repreſentation of an apparatus with the inclined planes and the picture.

The plane RTB is the picture, placed perpendicular on the ground plane ERTZ; ODE is the directing plane, and O is the ſpectator's eye, DVGK an original inclined plane, RF its interſection with the picture, DV its directing line, and DRK its interſection with the plane ERTZ; OVSF a plane that paſſes through the point of ſight O, and is perpendicular to the inclined plane DVGK; FS its interſection of the picture, KT the interſection of the plane ERTZ with the picture, AH the vaniſhing line of that plane; C the center of it and of the picture, (by Def. 7.) the plane OAP paſſing through the ſpectator's eye, and parallel to the inclined plane given; cutting the picture in the line AB, gives AB for the vaniſhing line of the inclined plane DVRGK; a line drawn through the center of the picture, perpendicular to AB, cutting that line in S, gives S for the center of that vaniſhing line (by Theo. 1.) To meaſure the angle of inclination between the inclined plane and the picture, at the point F, on the line FG, ſweep an arch, in this ſcheme it is ſuppoſed of 70 degrees; then the complement of the angle of inclination of the plane DVGK to that of the picture is 20 degrees, which is equal to the angle COS, which has been demonſtrated in Problem V.

To make this applicable to practice, I have given the following Examples.

### EXAMPLE XXXI.

*Having given the center, diſtance of the picture with the interſecting and vaniſhing line of a plane the picture ſtands perpendicular on: to find the vaniſhing line (with its center and diſtance) of another plane, whoſe interſection and angle of inclination to the picture are given.*

Let C (Fig. 2. Plate XXV.) be the center of the picture, CO the diſtance of the picture, LH the vaniſhing line given of the plane the picture ſtands on, QR the interſecting line of the picture with the plane, RN the interſection of the inclined plane with the picture; (the angle of inclination to the picture is here ſuppoſed 70 degrees.) Through C draw CG parallel to RN, and equal to the diſtance CO; from C raiſe the indefinite right line CS; perpendicular to CG draw GS, making an angle SGC equal to the complement of the angle of inclination of the plane to the picture, which gives 20 degrees for the angle SGC. Through S draw a right line ASB, parallel to CG or RN; then ASB will be the vaniſhing line ſought, S its center, and GS equal to its diſtance.

EXAMPLE

## EXAMPLE XXXII.

*The picture being prepared as above it is required, to find the projectoin of an inclined figure whoſe inclination to the picture is deſcribed in the preceding ſcheme, and its ichnography on the ground plane is the right angled figure* FDE.

If the original plane be continued to the picture, the line FD will interſect the picture in the point R, and L being the common interſection of the two vaniſhing lines HL, (the horizontal line) and ASB; therefore L is the vaniſhing point of the common interſection of the two planes, and the line RL on the picture is the indefinite repreſentation of DF; the points *d*, *e*, *f* are the projections of D, E, F, H is the vaniſhing point of the line *d e*, and is found by making LOH a right angle (the original DEF being ſo); the ſide of the original of *d e l*, is perpendicular to the ground plane as KM; BH is the vaniſhing line of that plane, and B the vaniſhing point of the inclined lines *d l*, *f g*; draw *d*B, *f*B, on *e* and *k*; draw *e l*, *k g*, perpendicular to the ground, cutting *f* B, *d*B, in *g*, and *l*; the lines *f d*, *e k*, and *g l*, vaniſh in point L, and *g l f d*, is the inclined ſurface. If the line KR (Fig. 1. Plate XXV.) is perpendicular to the picture as the line LW, then the angle GFS will be a right one, and the vaniſhing line AB will paſs through the center of the picture; and when the ſide KR is parallel to the picture, then will the vaniſhing line AB be parallel to AH; and ſuch poſition we ſhall term, having one ſide parallel to the picture. But as a certain modern author has laid down as a general rule (in this treatiſe of Parctical Perſpective) to find the vaniſhing line of an inclined plane and oblique to the picture, by an angle given, which angle of inclination is ſet on the horizontal line, without having regard to the interſection that inclined plane makes with the picture, which is falſe; for the vaniſhing line of any plane muſt be parallel to the interſection of that plane with the picture (by Theo. 6.) therefore, to prevent the reader's thinking I am partial, I have given the following example taken from his Practice. Let E (Fig. 5. Plate IX.) be the eye of the ſpectator, C the center of the picture, CE its diſtance, HL the horizontal line, and *to* one ſide given of the inclined face, Any where apart draw AB (Fig. X.) parallel to the horizontal line HL, and draw CB perpendicular to AB; then make an angle at A equal to the angle of inclination, and draw AC. Continue *to*, to the vaniſhing point L; and from L draw LE to the eye; then at E make a right angle with the line LE, and then becauſe the ſide which lies upon the ground is a right angle ſquare, therefore H is the vaniſhing point of the two ſides *tx*, and *os*, and L is the vaniſhing point of the other ſides *to*, *xs*; from the vaniſhing point H, draw HV perpendicular to the horizontal line, continue the horizontal line towards J; from H ſet off HJ equal to the diſtance HE of the vaniſhing line HV; then from J draw JV, parallel to AC in Fig. X. which will cut HV in V, and give HV for the vaniſhing line of the perpendicular plane *tyx*; and by drawing a line through the points V and L, we ſhall have VL for the vaniſhing line of the inclined plane *tozy*: But, as I have ſaid before, the vaniſhing line of that inclined plane cannot be found without having the interſection of that plane with the picture; nor can the angle BAC be the angle of inclination wanted, as the plane is ſuppoſed to incline to the picture. But on examining the Fig. 1. Plate XXV. this will appear erroneous. When the vaniſhing line of an inclined plane, whoſe interſection with the picture is parallel to the horizontal line, as when the interſection of that plane with the ground plane is parallel to the ground line; in that caſe the vaniſhing line of that plane and the horizontal line are parallel to each other; then if the diſtance of the picture is ſet on the horizontal line, on that point you place the angle the inclined plane makes with the ground, it will anſwer both the apparatus and Dr. Brook Taylor's Theory.

N. B. If a vaniſhing line ASB, center of the picture and diſtance is given, the angle of inclination may be found by converſe of Ex. XXXI. Fig. 2. Plate XXV.

### EXAMPLE XXXIII.

*How to repreſent an inclined plane, whoſe inclination is given, having one ſide parallel to the picture.*

Let HL (Fig. 1. Plate XXVI.) be the *horizontal* line, S the *center*, or point of ſight, L the the point of *diſtance*; draw S O perpendicular to HL, draw LO parallel to Y the agnle of the inclined ſurface, cutting S O at O, through that point draw VU parallel to HL for the vaniſhing line of the inclined plane; ſet one foot of the compaſſes at O, and the other in the point of diſtance L, draw the arch LV; then V is the point of diſtance to it. When the picture is thus prepared; ſet GB as one ſide, on the ground line GF, draw GO, BO, and from V, draw GV, cutting BO at I; draw I K parallel to GB; for the bottom that lies flat on the groud, draw BS; then finiſh it by the perpendicular 1*a*. The ſquare *de*, *fg*, Fig. 2. is found the ſame way, *de* being given.

### EXAMPLE XXXIV.

*In this ſcheme is the perſpective of a priſm that has one ſide perpendicular to the ground, and reſteth on an inclined plane.*

Let HL (Fig. 3. Plate XXVI.) be the horizontal line, S the center, and the point L the diſtance, Z the original priſm, X the inclined plane; draw SO perpendicular to HL; ſet the priſm Z at the point of diſtance L, and draw LO parallel to the upper ſurface, and LV for the under, cutting VSO at O and V; through theſe points draw lines parallel to HS, and from V and O, with the diſtance L, deſcribe the arcs KL, LN; the ſurface of the wedge being parallel with the upper ſurface of the priſm, the point O is the vaniſhing point to both, and V the vaniſhing point to the under ſide of the priſm; for the perſpective of the plane X, proceed as in the foregoing example, Fig. 1. Let *nl*, on the perſpective plane X be given (parallel to HL); through *n l*, from V, draw lines at pleaſure, from N the point of diſtance on the vaniſhing line NV, and through *l* draw a line cutting *na* at *a*;

make *a b* parallel to *n l*; from *a* and *b* draw *a* O, *b* O; either draw a perpendicular on *l* to find the point *d*, or (as the face Z repreſents a ſquare) the line K*a* and *ad* will be the diagonal of a ſquare; make *d c* parallel *a b*, cutting *a* O in *c*; then the figure is finiſhed. Fig. 4. is the ground plan of a ſquare (whoſe poſition is oblique to the picture) on an inclined plane, *b d* a line given. From I and K, the two points of diſtance, draw lines through *b* and *d*, interſecting each other at *a* and *c*.

## EXAMPLE XXXV.

*In this example are ſtairs put in perſpective, having one ſide parallel to the picture.*

Make NL (Plate XXVII.) the *horizontal* line, S the *center*, N the *diſtance*, I the given inclination, V the vaniſhing point of the inclination found, as by the foregoing example. Let 3 4 on the ground line be the depth, and FE the height of a ſtep; draw FS, ES and 3N, 4N, cutting ES at 2 and 1; then raiſe perpendiculars cutting FS at 5 W, for the firſt ſtep of the flight K; then drtw 5 V, WV, the point *d* is found by drawing a perpendicular from W, interſecting 5 V at *d*; then draw *d*S interſecting WV, and ſo on for as many ſteps as you pleaſe. The rails of the flight C vaniſhes at V, and the top of the landing-place at S; and each ſtep having one ſide perpendicular to the picture, the points *k*, *q*, *p*, &c. are found as the point *d*, the ſtairs N that runs below, are found by drawing a line from V through Y, and proceed as with the firſt ſtep K. X is a ray of light, and parallel to the picture; 6, 6, *r*, *r*, ſhadow of the rail D on the ſtairs.

## EXAMPLE XXXVI.

*In this example is an inclined chair in perſpective, having one ſide parallel to the picture.*

Let SQ (Fig. 1. Plate XXVIII.) be an *horizontal* line, CD a *ground* line; make USN perpendicular to SQ, S the *center*, Q the *diſtance*, QU a vaniſhing line for the ſide KA, or inclination above the horizon, NU the vaniſhing points, QN the vaniſhing line for the ſide DA, CB, &c. or reclination below the horizon: through the point N and U draw lines perpendicular to NSU, on them ſet P and R as points of diſtance, P equal to NO, and R equal to UO, C2; 1D are the thickneſs of the legs; to find the plane ABCD, draw NCB, NDA; then from P draw a line through D, cutting BC at B, make BA parallel to CD; for the ſide BLKA, which is a ſquare, draw BU, AU, and BR, interſecting AU at K, draw KL parallel to AB. For the ſide ML, CB, draw CU and LN cutting CU at M; the rail *b*5 is parallel to CD, and the ſides vaniſh at N. The diagonals FG, JP, interſect at O, for one of the rails; FP, another, &c. The length of this chair is known by the number of diagonals, every two diagonals forming a ſquare, whoſe ſide is equal to AB. Fig. 2. is a chair finiſhed.

## EXAMPLE XXXVII.

*To put an inclined cube in perſpective, whoſe poſition is oblique to the picture.*

Let HL (Fig. 1. Plate XXIX.) be the *horizontal* line, S the *center*, the perpendicular SO the *diſtance*, and *ac* a given ſide of a cube in the picture: continue *ac* to the horizontal line at H; draw HO and LO perpendicular to it; then H and L are two vaniſhing points. L for *de*, H for *bd*, *ac* and *fg*; through L draw CLA perpendicular to HL, then with one foot of the compaſſes in the point L, and with the opening LO, deſcribe the arch OC; make AL equal LO, and C will be the vaniſhing point of *df*, *ac*, *bg*, and A, of the ſides *ad*, *ef*, *cb*; draw HA, for the vaniſhing line of the ſides *ac*, *bd*, &c. draw SB perpendicular to AH. To find the point of diſtance E of that line, divide HA in two equal parts in G; ſet one foot of the compaſſes in G, with the opening GH deſcribe the ſemicircle HEA; through B draw a perpendicular line to HA, cutting the ſemicircle in E; join AE,

HE,

HE, which form a right angle, as the original. Through the given line *ac*, draw A*ad*, A*cb*, at pleaſure; at the point E ſet off 45 degrees (that being the angle, the diagonal makes with the ſide of a ſquare); at *b* and through *b*, draw E*b*D; from D, through *c*, draw a line cutting *ad* in *d*, and draw *d*H, interſecting *cb* in *b*, draw *d*C, *b*C, *a*C, and *d*L, the diagonal, cutting *a*C at *e*; draw A*e* at pleaſure, cutting *d*C in *f*, and draw *f*H, interſecting *b*C in *g*; ſo will the cube be finiſhed. If the diagonal *dg* is required, find *k* on CH, as the point D was on AH. For a demonſtration of this, ſee Problem XV.

## EXAMPLE XXXVIII.

*This example is of an inclined chair in perſpective.*

The lines points (Fig. 2. Plate XXIX.) ſtanding as in the foregoing example, the diagonals are obtained by E, D and L; *a b*, *c d*, *e f g*, are found as the cube; the back of the chair is three times its width, as appears by the diagonals *d g*, *i h*, *k t*, which are drawn to the point E.

## EXAMPLE XXXIX.

*To divide any right line in the picture that ſhall repreſent as many equal parts as are requird.*

Let KO (Plate XXX.) be a right line to be divided, (in this example KO repreſents one ſide of a ladder inclining on the ground, but perpendicular to the inclined plane KM); HL the horizontal line, C the center of the picture, NC the diſtance LB the vaniſhing line of the inclined plane, and V the vaniſhing point of lines perpendicular to that plane (found as by Prob. XII.) and of the line OK, draw VS and RP, parallel to it; then draw SOP, SKR; ſo that we may take RP for the original of KO; on PR ſet the diviſions 1, 2, 3, &c. &c. and through each of thoſe points draw 1S, 2S, 3S, cutting OK in *a*, *b*, *c*, &c. when the line OK is ſo divided, L being the vaniſhing point of *m*K, draw L*a*, L*b*, &c. which will repreſent every round of the ladder. If it is required the ladder W*t r* ſhould be of equal width with K*m*, L being the vaniſhing point of K*m*, make LI equal LN, draw I*m* cutting AK (which is made parallel to LI) in A; then is AK made as original of *m*K; produce AK to *lk*, and draw *l*C, *k*C; then any lines parallel to *lk*, between the lines *l*C and *k*C, repreſent lines equal to K*m*; N is the vaniſhing point of the ſide W*t*, and WT made parallel to NC draw CT*t*, and CW; on WT ſet the diviſions of equal parts, as 1, 2, &c. and proceed as you did with OK, only in this every round is drawn parallel to HL or *lk*. For a demonſtration of this example, ſee Prob. III.

## CHAP. IV.

### *Of ſhadows in general.*

Shadows are only the projections of given figures on given ſurfaces, by means of given luminious bodies; but to avoid the difficulties that would attend the deſcription of ſhadows, if the magnitude of the luminous body were taken in conſideration, it being ſufficient for practice to regard only the center of it, therefore, in theſe examples, a luminious body will be conſidered as a point. In ſhadows occaſioned by the ſun, the rays may be conſidered as parallel, becauſe they come from a point at an infinite diſtance from the picture; but thoſe from a ſmall light, as a torch, cannot be parallel, becauſe they come from a point not very diſtant from the picture.

To determine the ſhadows of bodies upon any plane whatever, it is ſufficient to know the ſhadows of the lines which are their bounds; and as the ſhadows we are now treating of are from the ſun, we ſhall here give rules to determine thoſe ſhadows, when the ſun is out of the plane of the picture, and when he is in the plane of the picture. For this end let the picture be prepared in this manner: ſuppoſe the ſun in R (Fig. 1. Plate XXXI.) and one of its rays to be RN, the plane FHG a picture perpendicular on the ground, O the ſpectator's eye: a line drawn parallel to RN, and paſſing through the ſpectator's eye, meeting the picture at the point T, which we ſhall call the *place of the ſun in the picture*, becauſe, as we have ſuppoſed the rays of the ſun parallel, determines the point T the vaniſhing point of the rays of light, in the picture; the point S on the ground is the ſeat of the ſun, and E the ſeat of the ſpectator's eye on that plane; a line paſſing thro' thoſe two points will cut the picture in *m*. Thro' *m* draw the line *m*M, perpendicular to the horizontal line HL cutting HL in M, for the vaniſhing point of the ſhadows on the ground; OT being drawn parallel to RN, will cut *m*M at point T, as has been mentioned before. The line MT*m* ſhews the ſun's rays are not perpendicular to the picture, for if they had, the line MT*m* would be removed to the line VG, and the point M would coincide with the point V, the center of the picture; therefore *m*TM is the vaniſhing line of the plane of rays SRNE. We ſhall call the point T, the point of inclination of the rays of the ſun (becauſe that point depends on the height of the ſun above the horizon, it being certain that point T will be nearer to the horizontal HL when the ſun is nearer the horizon); and the point M the point of declination of the rays of the ſun (becauſe that point depends on the declination of the ſun from the vertical plane OEVG, it being certain that it would be the line VG, in which caſe the two points *m* and G would coincide, and alſo both of them coincide in the center of the picture V, if the ſun was in the interſection of the vertical plane of the horizon; and they would wholly vaniſh if the ſun was in the zenith.) Obſerve that when the ſpectator is between the ſun and the picture, as in this figure the point T is below the horizontal line; but when the picture is between the ſpectator and the ſun; then point T is above the horizontal line, as in Fig. 2.

### *Rules for the ſhadows cauſed by the ſun on a plane perpendicular to the picture.*

KHT, (Fig. 2. Plate XXXI.) repreſents a picture, O the ſpectator's eye, R the ſun behind the picture, RN one of its rays, and SK the declination of the rays, CBD an original plane perpendicular to the picture, AB a line perpendicular to that plane. To prepare this picture, firſt draw Y*m* parallel to KS, cutting the picture in *m*; and through *m* draw the line *m* MT, perpendicular to the ground; then draw OT parallel to RN, meeting MT in T, for the ſun's place in the picture; let HM be the horizontal line, and OV, being drawn perpendicular to the picture, V, is the center of it; through V draw VI perpendicular to HM, for the vaniſhing line of the plane CBD; and draw TI perpendicular to VI, interſecting it in I: I is the ſeat of the ſun on the vaniſhing line VI, as M is the ſeat of it on the horizontal line. Then I is the vaniſhing point for the ſhadow on the vertical plane, as M is for the ſhadow on the ground; for BA is perpendicular to the plane CBD, and VI the vaniſhing line of that plane; therefore a plane of rays parallel to BA paſſing through the point of the ſun, will cut the picture in the line TI for the vaniſhing line of thoſe rays; and I being the common interſection of thoſe two vaniſhing lines, I is the vaniſhing point of the common interſection of thoſe two places. The little ſcheme, Fig. z. ſhews the picture with thoſe lines and points, and where the lines are parallel and perpendicular to one another: the other large ſcheme being ſeen ſide ways, lines that are ſaid to be parallel or perpendicular on the picture, as they do not appear ſo, might not be ſo well underſtood, therefore I thought that making the little ſcheme (which repreſents the large in a front view) would be more intelligible.

The ſcheme Fig. 3. Plate XXXI. ſhows how to find the ſhadows cauſed by the ſun on a plane KX, perpendicular to the picture, and the horizon; and alſo on a plane BCF, inclined on the horizontal plane, but perpendicular to the picture. *The ſun in this figure is ſuppoſed behind the picture.* Let HIM repreſent a picture, O a ſpectator's eye, R the place of the ſun, RN a ray of light, ST the declination of the rays to the picture, HM the horizontal line, W the ſeat of the eye on the ground, S the ſeat of the ſun alſo on the ground plane, and TL the interſection of the picture with the ground; draw WL parallel to ST, meeting the picture in L; draw L*r* perpendicular to TL, and O*r* parallel to the original ray RN, meeting the picture in the line L*r* in *r*, for the ſun's place in the picture; draw OV perpendicular to the picture, cutting it in V, for the center, then draw IV perpendicular to HM for the vaniſhing line of the vertical plane KX; and through V draw JV*v* for the vaniſhing line of the inclined plane BCF. Now to find on the picture a vaniſhing point for the ſeat of ſhadows on the plane KX, to expreſs the ſhadow KX, of KG, thro' *r* draw *r* I, perpendicular to IV, then I is the vaniſhing point of the ſeat of ſhadows on the vaniſhing line IV, to expreſs the ſhadow *kx* of the line *kg*; and *r*L is the vaniſhing line of the plane of rays (STRN); and M the vaniſhing point of the ſeat of ſhadows on the ground, that point being the common interſection of the vaniſhing line *r*ML and the horizontal line HM; if *r* I is continued to the vaniſhing line JV till it meets it in J, J will be the vaniſhing point for the ſeat of the ſhadows (BC of BA on the inclined plane) and expreſſes the ſhadow *bc* of *ba* on the picture; if *rv* is continued till it meets JV*v* in *v*, *v* will be vaniſhing point of the ſeat of the ſhadow EF of DE on the plane BCF and expreſs the ſhadow *ef* of the line *de*, in the picture, for planes of rays parallel to DE, and BA paſſing through the point of the ſun in *r*, will give J*r* and *rv* for the vaniſhing lines of thoſe planes; therefore the points J, I, *v* are the proper vaniſhing points of the interſection of thoſe planes and rightly found. The ſmall ſcheme next the large one, is the ſame, only ſeen in front.

To come to the practice, it will be neceſſary to have in the picture four chief points, the center or point V, Fig. 3. Plate XXXI. upon the horizontal line HM; the point M on the right or on the left, nearer or farther from the point V, according as the ſun declines more or leſs, on the right or left of the vertical plane; the point $r$ (where the picture is cut by a ray drawn from the center of the ſun and through the eye,) of the place of the ſun in the picture, which is called the vaniſhing point of the rays of the ſun, becauſe the rays of the ſun are ſuppoſed parallel to one another; this point $r$ will be found upon the line M $r$, perpendicular to the horizontal line; upon the vaniſhing line VI of the vertical plane is placed the point I, of the inclination of the rays above or below the horizontal line MH, and nearer to or farther from it, according as the ſun is ſuppoſed before or behind the picture; and higher or lower above the horizon. (Dr. Brook Taylor terms that point, the vaniſhing point of the ſeat of ſhadows on that plane.)

Obſerve, that to find the projection of any ſhadow, you muſt have a point in the picture for the luminous body, another perpendicular from that on the plane the ſhadow is to be caſt on, which we call the ſeat of light on that plane, with the point of the ſeat of a point propoſed, whoſe ſhadow is to be found on the ſame plane. The ſeat of the ſun's rays is found by letting a line fall perpendicular from the luminous point on any plane, as S (Fig. 2. Plate XXXI.) is the ſeat of the ſun placed at R; and in the picture, M repreſents that point, becauſe T is the ſun's place in the picture, and M is perpendicular from it on the vaniſhing line of the ground where S is on; I is the ſeat of T on the vaniſhing line of the vertical plane, for the ſame reaſon. If the line TL be continued as in Fig. 3. Plate XXXI. and a vaniſhing line $v$ V J is made on the picture paſſing through the center (that vaniſhing line will repreſent a plane, though inclined on the gound, is perpendicular to the picture) where it cuts it in J that point is the repreſentation of the ſeat of ſhadows on that plane, above the horizontal line; and if $r$M is continued, will give $v$ on $v$ V J for the ſeat on the ſame plane, below the horizontal line. When the planes lie oblique to the picture, the letting fall the perpendiculars for finding the ſeats will be taken otherwiſe, as will be ſhewn in its proper place; but when the rays are ſaid to be in a plane parallel to the picture, there need be only a line made to ſhew the inclination, or the angle it makes with the ground, as in the next example.

## EXAMPLE XLI.

Let HL (Fig 1. Plate XXXII.) be the horizontal line, C the center, and R a ray of light, ſuppoſed to come from the ſun, parallel to the picture; from each corner of the Figure A at top draw lines parallel to R, as 1 4, and 2 3; from their ſeats *a* and *b* draw lines parallel to the ground line, interſecting them at 4 and 3, or elſe find only the point 4, as C is the vaniſhing point of the ſide *a b*, 1 2; draw 4C, and from *b* draw a line parallel to the ground line, cutting 4C in 3.

The ſhadow *f e g* of the priſm B is found in this manner: L is the vaniſhing point of the ſide *c d*, make *f c* parallel to the ray R; from the ſeat of *c* at *a*, draw a line parallel HL, interſecting *c f* at *f*; draw *f* L, and from *g* the ſeat of *d*, draw a line parallel to *a f*, cutting *f* L at *e*.

To find the points *b*, *c*, *d* of the ſhadow of the inclined croſs D, From C through 4, draw a line at pleaſure; at 2 draw a line parallel to 1 4, cutting 4C at *a*; from the point 2 draw a line parallel to R, and from *a* a line parallel to HL, cutting it at *b*; the point *c* is found by drawing 1 *c* parallel to 2 *b* or R, and 4 *c* parallel to *a b*; the point *d* is found the ſame way.

## EXAMPLE XLII.

*In this example the light comes from behind the picture.*

HL (Fig. 2. Pl. XXXII.) the horizontal line, O the center; on HL at the point H raiſe the perpendicular RH, R is the vaniſhing point of the rays of the ſun, and H is the vaniſhing point of the ſhadow on the ground; to find the points A, B, C, of the ſhadow of the ſolid figure D, draw a line from R, through the point 1, at pleaſure; and through *a* (the ſeat of *t*) and H draw a line interſecting R 1 at A; the points B and C are found the ſame way. Obſerve, as O is the vaniſhing point of the ſide 2 3, it is alſo of the ſhadow BC; the ſhadow of the croſs is found in the ſame manner; and the ſhadow of it over the block S is found, by continuing that of the body of the croſs on the ground till it interſects the ſide *r* of the block at *s*, and from the interſection draw *s t* parallel to the upright of the croſs till it comes to the ſurface *t*; and from that point and H draw the ſhadow over it, (as it runs parallel with that on the ground), every point is found as in Fig. D. The ſhadow of the cylinder is found by ſetting off as many points on it at pleaſure, as *a*, *b*, *c*; find their ſeats on the ground, as 5, 6, &c. then proceed the ſame as with the ſolid figure D, 5 is the ſeat of *c*, and 1 is the point of its ſhadow, &c. The other points 2, 3, 4, are all found in the ſame manner.

### EXAMPLE XLIII.

*In this ſcheme the light from the ſun is before the picture.*

HV (Fig. 3. Pl. XXXII.) is the horizontal line, V the vaniſhing point of the ſhadow on the ground plane, L the vaniſhing point of light coming before the picture; the point 3 the ſhadow on the ground of the point *b* of the ſtairs, is found by drawing a line through the ſeat *a* to V, and another from *b* to the point L, interſecting *a* V at 3, the point ſought. The points 1, 2, are found in the ſame manner; *b g*, vaniſhes at C, as does the ſhadow 3 4. The ſhadow on the ſtep 3 *b c* is found by drawing 3 V; the ſhadow of the ſtick is projected by drawing AV, till it meets the bottom of the ſtairs; make the ſhadow *e* parallel to the ſtick where it meets the top, continue it towards V, the dotted lines from the head of the ſtick to the point L terminates the ſhadow of it on the flat top of the ſteps. The dotted lines of Fig. K, ſufficiently explain it; the ſide *ed* is parallel to the horizontal line, ſo is its ſhadow.

## EXAMPLE XLIV.

*In this example the light is ſuppoſed before the picture.*

V (Fig. 1. Plate XXXIII.) is the vaniſhing point of the ſhadow, W the vaniſhing point of rays of light; at any point on the quarter round where you want to project a ſhadow, make the ſection 3 4, as by Example XXVI. The point 4, ſhadow of *b*, is found by drawing *b*V cutting the quarter round at 3, there make a ſection; draw *b*W interſecting that interſection at point 4; point 2, ſhadow of *a*, is foundthe ſame way.

## EXAMPLE XLV.

XS (Fig. 2. Pl. XXXIII.) is the horizontal line, X the center, S the vaniſhing point of the ſhadow on the ground, R that of light coming from before the picture, CAB is a ſquare board placed on the top of a cylinder, *a b c* the ſeat of ABC, on the the ground, and 1 2 3 4 its ſhadow. The point J, the ſhadow of A on the cylinder, is found by drawing A S, cutting the top of the circumference, and from that point let fall a perpendicular; draw AR, interſecting it at J, the point ſought; ſet as many points on ABC, as may ſerve to complete the ſhadow, and proceed with every one in the ſame manner. E is the ſhadow of *e*, and *d* its ſeat on the ground; draw *d*S interſecting the bottom of the cylinder at D, there raiſe the perpendicular DF at pleaſure; draw *e*R cutting it at E, the line *d*DE is the ſhadow of the rail *de*, and the points G, F, &c. are found as point E. The ſcheme is ſufficient to explaine the reſt, lines projecting the ſhadows being on the plate.

## EXAMPLE XLVI.

In this ſcheme C (Fig. 3. Plate XXXIII.) is the center of the picture, H the vaniſhing point of ſhadow on the ground, and L the vaniſhing point of rays of light. To find the points *q*, S, *y*, ſhadow of K on the cylinder, from any point 2 on the circumference of the baſe of the cylinder (which is parallel to the picture) draw 2C, and from its ſeat $g$ draw $g$C; then from the ſeat of K draw lines to H cutting $g$C in $f$ and $u$; make $fy$, $uy$ perpendicular to the ground line, cutting 2C at $q$ and $y$, the points ſought; to find S take any point $r$, and proceed as with the point Q; any points $n$, $m$, of the ſhadow of the circumference of the inward cylinder on its ſurface, are found thus; draw CL, and draw $a\,t$, $b\,l$, parallel to it, cutting that circumference at $a, t, b, l$; then drawing $a$L, and $t$C, meeting in $n$, and $b$L, $l$C meeting in $m$, $n$, $m$, are the points ſought, $b$ being the ſeat of the point 1 on the outward circumference; draw 1L, $b$H, and their interſection is the ſhadow of the point 1. Fig. 4. is a hollow cylinder cut open, every thing remaining as in the foregoing example: $ar$, $bs$, $ot$, are parallel to CL; draw $r$C, $s$C, $t$C, and $a$L, $b$L, $c$L, interſecting each other at $e, d, f$, ſhadow of $abc$, 5 is the ſeat of point 3; by drawing 3L and 5H interſecting it at 2, give 2 for the ſhadow of point 3 on the ground.

## EXAMPLE XLVII.

In this ſcheme VH (Fig. 1. Plate XXIII.) is the *horizontal* line, V the *center*; make RS perpendicular to VH, S the vaniſhing point of the ſhadow on the ground, R the vaniſhing point of rays of light. Then in the circumference of the arch ONB (which is parallel to the picture, and vaniſhes at V) taking any point O, and finding its ſeat A on the ground, draw OR, AS, interſecting at $o$, the point ſought; the point $n$ on the wall EKBC, ſhadow of N, is found by drawing through P (the ſeat of N on the ground) PS cutting EC, (the ſeat of the wall) in $p$, draw NR, $pn$ parallel to DL or EK, interſecting NR at $n$. In the ſame manner are found all the points that form the ſhadow of the arch: *look for the point* R *at the bottom of the plate.* The points $d, c, f$ in Fig. 2. are found in the ſame manner as the point $n$ in the foregoing example; W is the vaniſhing point of ſhadows on the ground, and A the luminious point in the picture. For want of room in the ſcheme, both theſe points are placed in Scheme 3.

## EXAMPLE XLVIII.

### *How to find the projection of ſhadows in the cavity of a niche.*

Let ABCD (Plate XXXIV.) repreſent a nich parallel to the picture, HL the horizontal line, R the center, DB the diameter of the circular part of the nich, V the ſun's place in the picture, and H the vaniſhing point of the ſhadows on the ground, (or on any plane parallel to it.)

Firſt find the perſpective of the arch DKB, which is a ſection parallel to the horizon, and on DKB find the ſections that are perpendicular to it on every point wanted; then to find any point of ſhadow, ſuppoſe the ſhadow of B, which falls in the ſtraight part of the niche, draw from A (the ſeat of B) the line AH cutting the arch A*ag* in *a*; at the point *a* draw *a*6 perpendicular to HL; then draw BV cutting *a*6 in 6, for the point of ſhadow ſought; join A*a*6 by right lines for the ſhadow of AB. When the ſhadow ſought falls within the hollow of the ſphere DCBK, ſuppoſe it is required to find the point 2, the ſhadow of I; let fall a perpendicular line from I to interſect the line A*i* in *i*, for the ſeat of I; through *i* draw *i*H cutting A*ag*, in *l*; draw *l*K perpendicular to HL, meeting the arch DKB in K; from the point K draw the arch K2C, which is part of a circle in perſpective, which we call a ſection, perpendicular to the diameter DB; then draw IV cutting that ſection in point 2, for the ſhadow of I, as was required. By finding as many points as are thought moſt convenient the whole ſweep of the ſhadow 1 2 3 4 5 6, will be found. To render this clear I have left ſome lines, with points on the arch DCB marked E, F, G, and their ſeats marked *e*, *f*, *g*, which I think is ſufficient. The ſections may with a little practice be eaſily formed without the perſpective rule; only obſerving here, as the ſections come nearer to the arch DC, they are nearer that ſhape. Q*p*C*c* repreſents the ſhadow of QP*c*; the point on the croſs-bar PQ are left with the lines that give their ſeats, their interſection with the arch A*ag*, and their point of ſhadow are every one found as 6 the ſhadow of B.

## EXAMPLE XLIX.

In this ſcheme R (Fig. 2. Pl. XXXV.) is a ray of light from the ſun parallel to the picture, V the center of the picture, 1 6 4 5 a ſection of a ſphere parallel to the ground plane, 1 2 3 4 another perpendicular to the line R (as the rays from the ſun are to be conſidered as parallels, one half of the ball is enlightened.) The ſhadow of *p n l t* on the ground is found by drawing thro' every point of the circle 1 2 3.4 lines parallel to R 1, as V 3, P P; and lines parallel to the horizon, paſſing through the ſeats, will interſect them at *p n l t*. The little circle on the ground is the ſeat of the great one 1 2 3 4, and marked is by the ſame figures.

In this ſcheme the light comes from the candle L, (Fig. 1. Plate XXXV.) and L is the point of light; K its ſeat on the ground, and C the center of the picture; from K draw a line parallel to the ground line, cutting the ſeat of the ſide of the room TPX to T; draw TP parallel to KL, and LP parallel to KT, cutting TP at P, the ſeat of light on that ſide the room; produce TP to X; make LV parallel to PX, and XV parallel to LP; the point V is the ſeat of light on the cieling; W is the ſeat of light on the farther end of the room, and is found by drawing CK, and where it cuts the interſection of that ſide of the room and the floor raiſe a perpendicular, and draw CL, interſecting it at W. Having the different ſeats of light, proceed in projecting the ſhadows, which are found in the ſame manner as from the ſun. Point *b* ſhadow of *d*, is found by drawing through K, and the leg of the table, a line at pleaſure, interſecting it by L*b* at *b*, and every point of ſhadow on the ground of the table is found the ſame way; point *p* the ſhadow of *y*, is found by drawing L*p* at pleaſure, and through the ſeat of *y* and P, draw a line cutting L*p* at *p*; in this manner you may finiſh the ſhadow on the wall, of the circle I*y*. To find the point 8 on the ſide of the room, thro' K and 4 draw a line cutting the ſeat of that ſide of the room at *t*; and draw a line through L and the top of the chair at pleaſure, then draw 7 8 perpendicular to the ground line interſecting it at 8. The points 1, *h*, *k*, on the cieling, ſhadow of the croſs *e g f*, are found by drawing through the ſeat of *e*, *g*, *f*, and W, lines cutting the interſection of the cieling at *n*, (the croſs being perpendicular to the picture, its vaniſhing point is C); then through *n* draw C *n h*, and through every interſection for the points *i*, *k*. The ſhadow of the ball on the cieling is better underſtood by Fig. 3. *l* is the point of light, and S its ſeat, C the center of the ball; draw C*l*, divide it in two equal parts in *k*; ſet one foot of the compaſſes in *k*, with the opening *k*C cut the circumference of the ball in E and F, draw the line E*x*F. On the diameter 3 C 4, make the circle 1 2 3 4, parallel to the ground plane, and make another paſſing through the points 1 and 2, and the line C*k*; through the points *x* and V draw a line cutting the circle 2 6 5 1, at 5 and 6, and make the circle E 6 5 F. That upper part of the ball is as much as can be enlightened by a light placed at *l*, or as much as can be ſeen by a ſpectator at that diſtance. Find the points *e*. 6, 5, *f*, ſeats of E, 6, F, 5; then from S the ſeat of light, and through *e* 5 6 *f*, &c. draw lines parallel to the horizon, interſecting them by lines thro' *l* and the points E 6, F 5, at *n*, *r*, *m*, *s*; join thoſe points for the ſhadow of the ball on the ground.

## EXAMPLE L.

*This ſcheme* (*Plate* XXXVI.) *is taken from Dr. Brook Taylor*; *and though it does not repreſent the ſame objects, the vaniſhing lines and vaniſhing points are the ſame.*

ABC is the horizontal line, C the center of the picture; B is the vaniſhing point of the edge EF, which the chair reſts upon, and the other edge parallel to it; D is the vaniſhing point of the ſide GH, &c. and DA, perpendicular to AB, is the vaniſhing line of the plane EGH: P and Q are the vaniſhing points of the ſides MK and KN, of the regular tetraedron, and conſequently PQ is the vaniſhing line of the triangle KNM: L is the light and J its ſeat on the ground. Having AE*g*, that is the interſection of the upright plane EGH with the ground, and conſequently *g*, where HG meets it in the interſection of the edge HG with the ground, AE cuts the edge SR of the wall SRX in R; and conſequently X, where RX and GH meet, is the interſection of the line GH with the wall. DL*l* is the projection of a line parallel to the original GH (becauſe drawn through the vaniſhing point D), and A*l* is its ſeat; wherefore *l* is its interſection with the ground.

Now the originals of L*l* and HG being parallel, they are in the ſame plane, viz. in the plane which makes the ſhadow of HG; but *lg* repreſents the interſection of that plane with the ground; wherefore *lg* continued, is part of the ſhadow; and drawing LG cutting it in *o*, *o* is the ſhadow of G; and *o*S is that part of the ſhadow of HG which is on the ground. Then drawing SX and LH cutting it in *h*, *h*S is the other part of that ſhadow againſt the wall.

Having drawn *gp* and DT both parallel to AB, D*g* and *gp* are the projections of two lines in a plane, whoſe vaniſhing line is DT; and T is the vaniſhing point of the common interſection of that plane with the plane of the triangle KMN (by Corollary 2. Thorem 7.) PQT being its vaniſhing line. Therefore drawing T*p* cutting *g*D in V, V is the interſection of the line *g*GHD with the plane of that triangle KMN. Then *lg* cutting MK in *r*, drawing *rt*V, *rt* is that part of the ſhadow of GH, which falls on the triangle KMN. Having thus explained the manner of finding the ſhadow of GH, the reſt needs no explanation. To find the ſhadow of the picture P, proceed in the ſame manner as with the chair; A is the vaniſhing point of the edge Z*c* (where it is ſuppoſed the picture would interſect the ground if it was continued), and *y* (which is found by continuing ZW, and B*y*.) is the vaniſhing point of the ſide ZW, *y*B is the vaniſhing line of the plane Z*x*W; ULY, drawn through *y*, is parallel to the original of ZW; and BY is its ſeat; wherefore Y is its interſection with the ground; *f* the interſection of ZY with the wall, BZ cuts the wall *xf*W in *x*; W, where *x*W and ZW meet, is the interſection of the line ZW with the wall, and *fk* part of the ſhadow of the picture on the wall.

The reſt is eaſily found by what has already been ſhewn; *b* is the ſhadow of *a* on the picture P; *d* its ſeat on the ground, J*dc* interſects Z*c* in *c*; draw *cy*, to cut L*ab* in *b*. Theſe ſhadows may be all found by finding the ſeat of light on the walls, and the ſeat of every line of the objects on the ground, as in the preceding example; but I have given this, as it carries along with it a clear demonſtration.

## CHAP. V.

### *Of finding the reflection of any figure on poliſhed planes or ſtanding water.*

### EXAMPLE LI.

It is well known, that the reflection of figures on a poliſhed plane, or on the ſurface of ſtanding water, appear to be juſt as much on one ſide of the plane, as the real objects are on the other ſide: ſo that to find the reflected repreſentation of any point, you muſt draw a perpendicular to the reflecting plane from the real point, and in it take a point at the ſame diſtance on the contrary ſide of the plane. For inſtance, to find the reflection of the perpendicular AB (Fig. 1. Plate XXXVII.) let AC be the ſurface of the reflecting plane, and A the ſeat of B; continue the perpendicular AB downwards till A*b* is equal to AB. The angle any original line makes with the reflecting plane, is the ſame as the reflection with the plane, as you ſee by the inclined line FD Fig. 2. DC being the ſurface of the water, the angle GDF is equal to the angle CDF of the reflection; O is the vaniſhing point of the ſquare board Fig. 3. which is parallel to the reflecting plane; therefore O is the vaniſhing point of its reflection, as you ſee in the conſtruction of that figure, which is ſufficiently clear by the ſcheme only.

EXAMPLE

## EXAMPLE LII.

In this example (Fig. 3. Plate XXXIII.) the ſurface of the water is ſuppoſed even with the ground, or the line *hg*, and the circles in the reflection are the ſame ſize as the real ones, and the ſame diſtance from the line *hg* downwards, C is the vaniſhing point, and every point of ſhadow in the reflection may be found in the ſame manner, as *n m* in the real figures making *l*H equal to HL, and uſing the point *l* inſtead of point L.

## EXAMPLE LIII.

In this ſcheme is ſhewn the manner how the reflection is found in an inclined looking-glaſs, of abird-cage hanging from the ceiling, as in Fig. 1. Plate XXXVIII.

A is the center of the picture, and AB the vaniſhing line of the ground; the diſtance of the picture being equal to AB, AC is the vaniſhing line of the ſide GHK of the bird-cage, and CD the vaniſhing line of the inclined looking-glaſs. Through *a*, where the edge *ba* of the leg of the table cuts the ſurface of it, drawing *ae* parallel to AB, and thro' *b* draw *bd* parallel to *ae*; *e* is the ſeat of the line GK of the ſide of the bird-cage on the ground. Then drawing A*c*, cutting the interſection of the ſurface of the ſide GHK of the bird-cage with the ground in *d*, and then drawing *de* parallel to AC, and cutting *ae* in *e*, drawing A*e*, is got the projection A*e* of the common interſection of the ſurface of the table and the plane GHK of the cage. For *ae*, being parallel to AB, is the projection of a line in the ſurface of the table parallel to the picture; and for the ſame reaſen *db* is the projection of a line on the ground, and *de* is the projection of a line in the plane of the ſide of the bird-cage, both of them parallel to the picture; *ab* is alſo the projection of a line parallel to the picture. Therefore *abde* is the projection of a trapezium parallel to the picture, whoſe angle *e* is in the common interſection of the ſurface of the table and of the ſide GHK of the bird-cage (if produced.) But A being

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the common interſection of the vaniſhing lines of thoſe two planes, is the vaniſhing point of their common interſection; and therefore *e* A is the projection of that interſection (by Corollary 2. Theorem 7.) For the ſame reaſon *o* being the projection of the point where the ſurface of the glaſs touches the table, and E being the common interſection of the vaniſhing lines AB and CD, *o*E is the projection of the common interſection of the ſurface of the table and the ſurface of the glaſs. Therefore *f*, where *o*E and *e*A meet, is the projection of the point where the three planes meet, of the ſurface of the table, the glaſs, and the ſide GHK of the cage. Therefore drawing *f*C, it is the projection of the common interſection of the ſide GHK and the looking-glaſs.

Having the vaniſhing point P of lines perpendicular to the plane of the looking-glaſs, whoſe vaniſhing line is CD (which is got by drawing AO parallel to DC and equal to the diſtance of the picture, then thro' A draw DP perpendicular to DC, join OD and make OP perpendicular to OD cutting DP in P), drawing PA thro' the vaniſhing point A (of the line GH) and cutting CD in D, D is the vaniſhing point of the ſeat of GH on the plane of the glaſs. Therefore GH cutting C*f* in *i*, D*i* is the projection of that ſeat. Then drawing GP (which repreſent a line perpendicular on the looking-glaſs) cutting D*i* in *k*, *k* is the ſeat of the point G on the plane of the glaſs: wherefore through the point G draw GQ parallel to AP, produce D*ik* to cut the line GO in *n*, make *n*Q equal to *n*G, and by drawing DQ the point *g* is got which is the repreſentation of the reflection of point G, and *gi* is the reflection of G*i*; and drawing PH cutting *gi* in *h*, *gh* is the reflection of the line GH: and in the ſame manner may be found any other lines in the reflection. The reflection of the ſide GHK may alſo be deſcribed by its vaniſhing line, for in PAD, making *a*D to repreſent a line equal to that of AD (which is got by drawing thro' P (of the line PAD) at pleaſure the line PO (Fig. 2.) and make *cb* parallel to it, then thro' DP draw OD, OA cutting *cb*, in *d* and *b* makes *cd* equal to *db*, and draw O*c*; then produce DP till it cuts O*c* in *a*; let the line PD, and diſtance AD, Fig. 2. be equal to that in Fig. 1. and transfer *a* on the line PD, Fig. 1. then *a* is the vaniſhing point of the reflected line *gh*, and C*a* is the vaniſhing line of the reflected ſide GHK.

## CHAP. VI.

### *Of the perſpective of landſcapes.*

### EXAMPLE LIV.

*To find the perſpective plan of a building, having the meaſures laid on the ground line.*

In this ſcheme, Fig. 4. Plate XXXVII. it is intended the perſpective plan *e*H*d* lies oblique to the ground line GD. AB is the horizontal line, C is the center of the picture, and OC the diſtance; make AOB a right angle, cutting the horizontal line in A and B, for the vaniſhing points of the ſides H*d* and H*e* (ſuppoſing their original lines to be at right angles.) Let GH and HD on the ground line be two ſides of the plan wanted in perſpective, and A the vaniſhing point of line *g*H; draw HA. To determine the point *g*, take AO for radius, with which deſcribe the arch OU, A being the vaniſhing point of H*g*, AU will be the diſtance; ſet any point G on the ground line, and draw GU cutting H*g* in *g*, the point ſought: in this manner may be cut any line in the picture; if B is the vaniſhing point of the ſide H*d*, F is the diſtance, BO being radius, and OF the arch; make HD equal to HG, draw HB and FD, cutting HB at *d*, then draw *d*A, *g*B interſecting at *h*, and H*ghd* is the perſpective of a ſquare, whoſe ſides are equal to HG or DH, and the angle AOB.

### EXAMPLE LV.

In this example every thing ſtanding as in the laſt, on the ground line ſet any meaſure given, as HD (Fig. 5. Plate XXXVII.) for

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the front of a building, IL for the center part of it, K the middle of the pediment. MG its depth, and MH the diſtance the corner of that building is from the picture. Draw HF, MU interſecting at *b*, draw *b*B, *b*A, and from the points I, K, L, D, draw lines to the point F, cutting *b*B at *i*, *k*, *l*, *d*, and from the points M, N, G draw lines to U cutting *b*A at *ng*. To give the proper height of every part of the building, produce *b*B to the ground line at P, make PQ perpendicular to GD, ſet on it the ſeveral dimenſions, and on *b*, *i*, *k*, *l*, *d*, *ng* raiſe lines perpendicular to the ground line, and thro' every diviſion on PQ, draw lines to B cutting the perpendiculars *b*, *i*, *k*, *l*, *d*, at *b*, *r*, *x*, and through the points *b*, R on *b*, draw lines to A, cutting the perpendiculars *n* at T, and *g* at V, join *b*TV for the roof. To find its vaniſhing point, make EA perpendicular to AB; produce *b*T, cutting EA in E for the vaniſhing point of the ſide of the roof *b*T*xu*. If EA be continued below the horizontal line, and TV produced, their interſection will be the vaniſhing point for TV. To finiſh the pediment *wfu*. draw *k*A, *n*B at their interſection *s*, raiſe a perpendicular to GD, draw TB, cutting it at *y*, then draw A*io*, A*rw*, make *ow* parallel to *ir*, draw *o*B, *w*B, draw a line thro' A and *l*, cutting *o*B at *z*; make *zu*, parallel to *wo*, cutting *w*B at *u*, through KF draw a line cutting *oz* at *e*, at that point raiſe a perpendicular to GD, and through *y* and A draw *yf*, cutting *ef* at *f*; then join *wfu*, draw *at* B (the top of the window) till it cuts *ir* at *t*, then draw A*tc* cutting *ow* at *c*, draw *c*B for the top of the window on that part of the building, proceed in the ſame manner for the under part of them; for the windows on the ſide F of the houſe, it is only uſing the point A inſtead of the point B.

## EXAMPLE LVI.

In this example is repreſented a landſcape that has a deſcent marked G*ab*1 (Fig 1. Plate XXXIX.) a riſing ground *ed*K*n*, and GC1 a flat country, HL the horizontal line, C the center, and

L the diſtance, BF the vaniſhing line of a plane inclined below the horizon, LB its inclination, B the center, and F the diſtance on it, AS the vaniſhing line of an inclined plane above the horizon, LA its inclination, A the center, S the point of diſtance on it; draw GB, IB, and GF, cutting IB in *b*, make *ba* parallel to GI. For the ſquare place of water, draw *a*C, *b*C; then *a*L, or *b*H, cutting *a*C at *c*, make *cd* parallel to *ab*. For the aſcent, draw *c*A, *d*A, interſecting *c*A by *d*S, at K make K*n* parallel to *cd*. GK*n*I being perpendicular to the picture, vaniſhes at C, and D is the vaniſhing point of the houſe E. To find the point *k* in the water (*ao* being the ſurface) draw the perpendicular K*k* at pleaſure, and *a*C cutting it at *r* (*r* is the ſeat of K on the water) make *rk* equal to *r*K; the other points are found the ſame way, *ck* is the reflection of *c*K, and *ks* of K*n*.

The ſhadows which are ſuppoſed to be caſt by the ſun, and parallel to the picture, the line R is the inclination of the rays of light, make *li* parallel to it, and *b*I parallel to the horizontal line, join I*ib*. For the ſhadow of I*l* on the ſlope, draw *im* to the point C, cutting *cd* in *m*, join *nm*, then I*imn* is the ſhadow of I*lm* on the down-hill ſurface of the water and riſing ground. The poſts T, U, and their ſhadows, vaniſh at B. Plate XL. will ſufficiently explain the reſt.

## EXAMPLE LVII.

In this example HL (Fig. 2. Plate XXXIX.) is the horizontal line, C the center (or the point of ſight removed in the picture) GD the ground-line, B a riſing ground, ND the ſeat of it, NB the perpendicular height at B, equal to DA on the ground-line, KNI being produced towards the horizontal line till it cuts it, that point is the diſtance; and if KN, GD, be continued till they meet, from their interſection to D will be the real diſtance between the point D and point N. The level ground at a great diſtance riſes to the horizontal line; the tops of the hills are higher, but their ſeats are below it. The light which is from the ſun is parallel to the picture, and the line R is the inclination of the rays of light: every ſhadow, and reflection in the water is found as in the foregoing example. Plate XLI. repreſents the ſame when finiſhed.

## EXAMPLE LVIII.

In this landſcape are repreſented ſeveral ſtreets, whoſe vaniſhing points are H, C, L (Fig. 1. Plate XLII.) placed on the horizontal line HL, C is the center of the picture; the houſes ſtanding obliquely with the picture, having one ſide vaniſhing at the point L, and the other at H; the houſe F, having one ſide perpendicular to the picture, vaniſhes at C; the ſtreets are ſuppoſed to be at right angle, with each other; O is an octagon baſon, the perſpective of it is found by the points H, *d*, *e*, L on the horizontal line, and *lm* as a diameter. On VH, perpendicular to HL, are ſet the vaniſhing points QS, the ſide of the roof vaniſhes at S, the ſhed C at Q, and the roof *v* of the houſe *e* at V; the oppoſite ſide forming the angle vaniſhes at a point as far below the horizontal line as the other is above it. Make *k*C perpendicular to HL, continue one ſide of the pediment *g* of the houſe F, cutting *k*C at *k*, the vaniſhing point for that ſide. If lines are drawn from the top and bottom of the houſe *e* to the point L, any line perpendicular to HL between thoſe lines, as 1, 2, will be equal to the height of the houſe *e*. Through 1 and 2 draw lines to H, for the top and bottom of the houſes in that ſtreet, the line RL is perpendicular to HL, and *b* is the vaniſhing point of *z* the roof of the church B. The light is parallel to the picture, and to the line RH; ſo that the ſhadow *x* of the point *g* is found by drawing 3*x* through the ſeat parallel to HL, and *gx* parallel to HR cutting it in *x*. All the rays being parallel to RH, and H being the vaniſhing point of *ta* (the ſide of the houſe before the chuch) HR is the vaniſhing line of the plane made by the rays which paſs through the line *ta*, and LR being the vaniſhing line of the church B; on which the ſhadow *tu* is caſt, R is the vaniſhing point of the common interſection of thoſe two planes, that is of the ſhadow *tu* of the line *ta*; therefore the line *tu* is drawn through the point R, and not parallel to HR as $g\ x$ for the ſhadow $3x$; through G the interſecting point of the two vaniſhing lines QL, RH, draw $us$ for the continuation of the ſhadow $tu$, of $ta$ on the top of the ſhed, and $s\ w$ perpendicular to HL finiſhes the ſhadow of $ta$ and the ſide of the houſe. Fig. 2. repreſents the ſame when finiſhed.

## EXAMPLE LIX.

In this landſcape the houſes A, B, D, E (Plate XLIII.) have one ſide perpendicular to the picture, which therefore vaniſhes at C, the center; the dotted lines drawn through the bottom of the houſes is their ſeat, (which is even with the ſurface of the water) make every reflection as far below that line as the real object is above it, as appears by Fig. 1, 2, 3, in the water anſwering the ſame figures above it; the point O is the vaniſhing point for the boards F and its reflection $f$; the light is ſuppoſed coming from behind the picture, O is the vaniſhing point of the ſhadow on the ground, but the vaniſhing point of the rays of light are out of the plate, to ſhorten the ſhadows. Plate XLIV. repreſents the ſame finiſhed.

## EXAMPLE LX.

*How to find what ſize an object is to be, that, when placed on any given height, and at any given diſtance from a ſpectator, it ſhall appear to him of the ſame ſize of another placed at a given diſtance.*

Let a ſpectator be placed at O, Fig. 1. Plate XLV. and a vaſe C placed from him on the ground at about 15 feet, it is required that another vaſe be made, that, when placed on a wall at about 50 feet, ſhall appear to him of the ſame ſize with the vaſe C. Draw two lines, one from the bottom the other from the top of the vaſe C, meeting at the ſpectator's eye O, making the angle AOB; then draw the line OF, to reach the bottom where the vaſe is to be placed on the wall; and make an angle at the eye $aOb$ equal to the angle AOB; and draw $Ob$, and the perpendicular FD cutting O*b* in D is the height the vaſe D, muſt be made to appear there the ſize of the vaſe C.

## EXAMPLE LXI.

*In this ſcheme Fig. 2. Plate XLV. is ſhewn how figures, or any other objects in a picture, ſhall be drawn ſo as to repreſent figures of the ſame ſize.*

Draw two lines from the top and bottom of any figure, which you ſhall place on the ground line (as ſuppoſe) from figure A to the horizontal line in V; if a figure ſhould be wanted at B, draw B*b* parallel to the ground line, cutting V*b* in *b*; then take the perpendicular *b c* for the height of figure B; figure D is found in the ſame manner and is equal to *d*. The bird *y t* is farther off, and is of the ſame ſize as the bird GH; for it is made equal to the line *g h*, which is parallel to GH; and between the lines GV and HV, and forms the ſame angle. In the ſame manner may you find how large a figure muſt be in front from one at a great diſtance; for ſuppoſe the little perpendicular *d* repreſented an object a far off, draw through V, the line CV and V*db*, paſſing through the top and bottom of that perpendicular, that at point *b*, an object *b e* will repreſent one like that placed at *d*.

## EXAMPLE LXII.

In this ſcheme, Fig. 1. Plate XLVI. the wall *tu*O, and the wall *t a b*, ſtand at right angles with one another, H is the vaniſhing point of the wall *t b a*, L is the vaniſhing point of the the wall *t u* O, and HL is the horizontal line; the ſun's rays are ſuppoſed to be in a plane parallel to the picture, and the inclination of the rays to be repreſented by the line RH, the ſides of the roof of the ſhed D vaniſhes at point Q, and QL is the vaniſhing line of that inclined plane. The ſhadow *t u c* on the wall *t u* O of the wall *t a b* is found thus: all the rays being parallel to RH, and H being the vaniſhing point of *t a*, HR is the vaniſhing line of the plane made by the rays which paſs through the line *t a*, and L R being the vaniſhing line of the perpendicular plane *t u* O on which the ſhadow *t u e* is caſt, R is the vaniſhing point of the common interſection of thoſe two planes, therefore through R *t* draw R *t u*, and through the point *a* draw *a u* parallel to R H cutting *t u* in *u*. Draw *b c* parallel to the horizontal line, cutting the edge of the wall in *c*; join *c u*, then *b c u* is the ſhadow of *a b*, part on the ground and part on the wall. The point *d* and any other of the top of the light on the wall *t u* O are found in the ſame manner.

The ſhadow on the ſhed D is found as follows: Let H L, Fig. 2. Plate XLVI. be the horizontal line, L the vaniſhing point of the ſide *t* B of the wall where the ſhed is againſt, whoſe ſide *w s* vaniſhes there likewiſe, and H is the vaniſhing point of the line *t a*, T repreſents a ray of light parallel to the picture, a plane of rays parallel to *t a*, inclined as the line T paſſing through the point H will give H R for the vaniſhing line of it, and L R being the vaniſhing line of the perpendicular plane *t* B *v* on which the ſhadow *t u* is caſt, R is the vaniſhing point of the common interſection of thoſe two planes, (being the common interſection of thoſe two vaniſhing lines) then draw R *t u*; *t u* in this example being only part of *a t*, to get the part wanting on the roof D, proceed thus; Q being the vaniſhing point of the inclined ſides of the roof, Q L is the vaniſhing line of that inclined plane D, where the vaniſhing lines H R, Q L interſects each other in G; that point G is the vaniſhing point of the common interſection of thoſe two planes, therefore where *t u* interſects (if need be) in *u*, draw G *u* for the continuation of that ſhadow on the roof; to find point *r*, the ſhadow of *a*, draw *a r* parallel to T or H R, cutting *u r* in *r*, then *t u r* is the ſhadow of *at*, partly on the wall B and the roof D; for the ſhadow of *a* A on the roof, you muſt draw Q *b* cutting *a* A in *b*, then draw *b s r*, then *r s* is part of the ſhadow of *a* A on the roof. To get the remaining part, draw A *w* parallel to H L, cutting the edge of the ſhed on the ground in *w*; join *w s*, then is A *w s r* the ſhadow of *a* A on the ground, the ſide of the ſhed, and on the roof as was required.

## EXAMPLE LXIII.

The ſcheme Plate XLVII. is the repreſentation of a thatched houſe BGN, having one ſide BNE perpendicular to the picture *j*J, paſſing through the center C, is the vaniſhing line of that ſide, and the line VCH, paſſing alſo through the point C, is the vaniſhing line of the inclined plane CA*a* of the houſe; becauſe that plane though inclined with the ground, is perpendicular to the picture, and the ſide G*a*K is parallel to it; the ſun's place in the picture is the point O (ſuppoſing the light coming from behind the picture); L is the vaniſhing point of the ſeat of ſhadow on the ground; *j* that on the vaniſhing line of the vertical plane DNE; V on the vaniſhing line of the inclined plane above the horizon; (as in Fig. 3. Plate XXXI.) Then to find the ſhadow E*d* of the line ED, through *j* draw the ſeat of the ſhadow E*d*, and through the point O and D draw O*d* cutting E*d* in *d*; then is E*d* the ſhadow of DE: the ſhadow A*a* of AB on the roof GA*a* is found by drawing through the point V and through A the line A*a*; then through O draw OB*a* cutting A*a* in *a*; then A*a* is the ſhadow of AB on the roof; for AB is parallel to OV, and OV is the vaniſhing line of a plane made by the rays paſſing through the line BA; the ſhadow GF of the chimney FI, is found by drawing HFG, and OI interſecting FG in G; for OH is the vaniſhing line of a plane, made by the rays paſſing through IF, and H being the vaniſhing point of FG, H is the common interſection of the vaniſhing lines OH, and VH, the vaniſhing line of the inclined plane of the roof where the ſhadow GF is caſt, and for the ſame reaſon V is the vaniſhing point of A*a*; the ſeat of the ſhadow P*n* of the line PN is found as the ſhadow AF. Thoſe on the ground, or on any plane parallel to it, are found by drawing through the ſeat of the object a line to the point L, and from the top another paſſing through the point O, to determine the ſhadow.

## EXAMPLE LXIV.

In this ſcheme, Plate XLVIII. is the repreſentation of a houſe that ſtands obliquely to the picture; L is the vaniſhing point of the ſide G, H is the vaniſhing point of the ſide TQ; and HL the horizontal line; the rays coming from the ſun are in a plane parallel to the picture, and parallel to the line VL; the roof of the houſe is pyramidical; A is the center of it, and is found by drawing the diagonals through the corners of the building; AO is the axis of it, and is perpendicular to HL; to find the ſides, draw QO, TO, GO, for the inclined ſides of the roof, and $w$H, $w$L to finiſh the top. To make the garret window $che$, draw $ac$ perpendicular to HL, and ſuppoſe $ab$ the height wanted for the hole made in the roof; draw $a$O, then $c$L, $b$L cutting $a$O in $d$ and $y$, let $a$ repreſent the middle of the window, and $xm$ the breadth; draw $xb$, $me$ perpendicular, cutting $he$ in $h$ and $e$; then draw $e$L, $h$L, and draw $d$L cutting $e$L, $h$L in $g$ and $f$, join $gx$, $gf$, $fm$, to repreſent the ſquare hole cut in the roof, $efm$ is the ſide that projects from it, and $hce$ the pointed top. To find the ſhadows that are caſt on this roof, proceed thus, continue TA, to the horizontal line in X, and draw XD perpendicular to HL, and produce the line TO to meet XD in D; X is the vaniſhing point of TA and XD the vaniſhing line of the plane TAO; D is the vaniſhing point of TO, and H the vaniſhing point of TQ; therefore DH is the vaniſhing line of the inclined plane TOQR; to find the ſhadow R (on the roof) of the point B, L is the vaniſhing point of the line BP, and VL the vaniſhing line made by a plane of rays paſſing through the line BP, Y is the common interſection of thoſe two vaniſhing lines, that is, that of the plane of rays and that of the plane the ſhadow is to be caſt on; therefore Y is the vaniſhing point of the common interſection of thoſe two planes; as the plane of the roof does not join with the plane BPQ of the chimney, but only in point Q, draw QPZ to meet the interſection of the vaniſhing line DH, LW, when produced, that point being the vaniſhing point of the common interſections of thoſe planes, for LW is the vaniſhing line of the plane BQP, where the line cuts BP in P; draw YPR and BR, parallel to VL, cutting PR in R; then PR is the ſhadow ſought.

The ſhadow *if* is found by drawing Y*fi*, and *ei* parallel to VL, cutting *fi* in *i*; to get the ſhadow *n* on the chimney, *tn* is part of the ſeat of the ſhadow of *tw*, find the projection of the chimney in the ſame manner as you did the window; then the line where the edge of the chimney and the roof meets, muſt be continued to the top of the ridge, cutting it in *t*; and through that point and V draw V*tn*, and if you draw a line thro' *w* parallel to VL, it will cut *tn* in *n*: The reaſon why the point V is uſed here, is becauſe VH is the vaniſhing line of that ſide of the chimney, and L the vaniſhing point of *tw*. As the chimney in the ſcheme is ſmall, I have inſerted the Fig. M, with the lines that conſtructs it; and the ſmall line 1, 2, is to find the middle of the front of the chimney; to divide lines in equal parts, ſee Prob. 3, and 4. The ſhadow of the ſign-poſt is drawn parallel to HL, and the croſs-bar and ſign drawn to the point L, and their ſhadows determined by a line parallel to VL.

## EXAMPLE LXV.

In this ſcheme, Plate XLIX. the houſe B ſtands obliquely to the picture; Z is the vaniſhing point of the ſide B*f*; H is the vaniſhing point of the ſide D*y*; ZH is the horizontal line, and C the center of the picture. To find the different ſhadows in this landſcape, proceed in this manner; the point R is the ſun's place in the picture, S the vaniſhing point of the ſeat of ſhadow on the ground, and SRN the vaniſhing line of the plane of rays. The ſhadow of the pole *d*G, is found by drawing *d*S to interſect the edge of the ſhed *hu* on the ground in *h*; as *d*G is perpendicular to the ground, draw *hu* perpendicular to HZ, till it cuts the edge of the roof of the ſhed in *u*; Z being the vaniſhing point of the ſide of the houſe B*f*; ZV is the vaniſhing line of the inclined roofs of the ſhed and of the houſe; where the vaniſhing line SRN cuts the vaniſhing line ZN in N, N is the vaniſhing point of their common interſection; having got the point N, draw N*u* along the top of the ſhed, till it cuts the edge of the roof of the ſhed and the wall in *o*; draw *ot* (perpendicular to HZ) to the top of the wall, but where it cuts the edge of the roof under the window in *q*, draw *q*N, and *e* is part of the ſhadow on the roof of the houſe, ſo that *dbuofqe* is the ſhadow of the pole, part on the ground, part on the ſhed, and part on the wall and roof of the houſe. To get the ſhadow *af*, Z*r* is the vaniſhing line of the perpendicular plane of the ſide B*f*, of the houſe B, and H is the vaniſhing point of the line *ba*, then HR*r* is the vaniſhing line made by a plane of rays paſſing through the line *ba*, for the original of *r*RH and *ba*H are parallel to each other, their projections paſſing through the ſame vaniſhing point H; then to determine the point *f*, draw *ra* and *b*R to cut it in *f*; the ſhadow *t* of the chimney is found by drawing from the ſeat of the chimney a line to the point N, and another from the top of it to the point R, interſecting it in the point *t*. To find the ſhadow D*y* of the line DT, DT vaniſhing at C, CS perpendicular to HZ, is the vaniſhing line of the perpendicular plane of the wall DT, and of the houſe A, whoſe ſide A*p* vaniſhes alſo in the point C, the point H is the vaniſhing point of the ſide D*y*, the ſhadow D*y* of the line DT is caſt on, and HVY is the vaniſhing line of the plane of the ſide D*y*: draw CRY cutting HY in Y, C being the vaniſhing point of DT, CY is the vaniſhing line made by a plane of rays paſſing through DT, and Y being the common interſection of thoſe two vaniſhing lines, that is, that of the plane of rays and that of the plane the ſhadow is caſt on, Y is the vaniſhing point, or the ſeat of the ſhadow of the line DT in that plane; therefore draw DY and TR to determine the point *y*; to get the ſhadow *np* of *no* on the wall A, C is the vaniſhing point of the ſide A*n* of the houſe A, and AS is the vaniſhing line of that plane, *n* ſuppoſed perpendicular on the vertical plane A, a line drawn through R perpendicular to CS, cutting the vaniſhing lines CS of the vertical plane of the houſe, the ſhadow *np* of *no* is to be caſt on, S is the vaniſhing point to the ſeat of the ſhadow *np*, therefore draw *n*S and *o*R to determine the point *p*.

If theſe laſt examples are well underſtood, the practioner will not find it difficult to execute any piece where ſhadows may fall on any ſurface; for it is only finding the vaniſhing point of the lines you want to find the ſhadows off, and the vaniſhing line of the plane where the ſhadows are to be caſt on, and then the vaniſhing lines made by planes of rays paſſing through thoſe lines whoſe ſhadows is wanted, and where the vaniſhing lines interſect each other, that point is the vaniſhing point for the ſeat of the ſhadow required.

Plate L. and LI. are two landſcapes; the firſt repreſenting a riſing ground in the front, and anſwers to Fig. 2. Plate XXXIX. the other a flat conntry, and anſwers to Plate XLIII.

Theſe examples are ſufficient for the young practitioner to draw landſcapes, views, &c. &c. in perſpective, if he has already acquired ſome ſkill in drawing, as by theſe rules he will be enabled to adjuſt and correct his deſigns, and be capable of perſoming them to the greateſt nicety and exactneſs.

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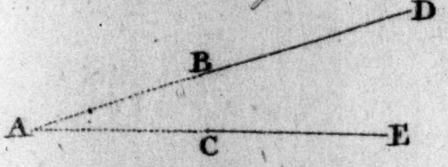
Fig: 1



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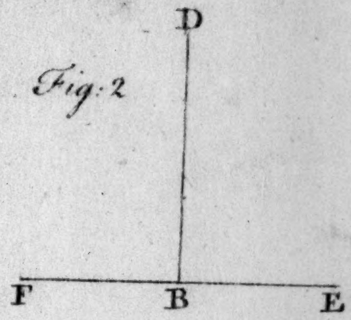

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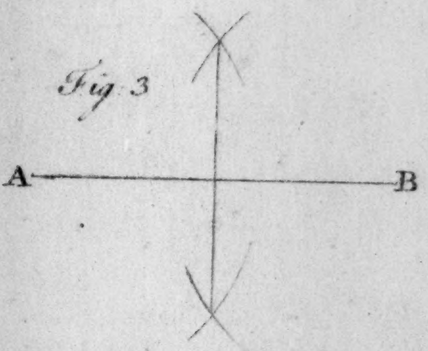

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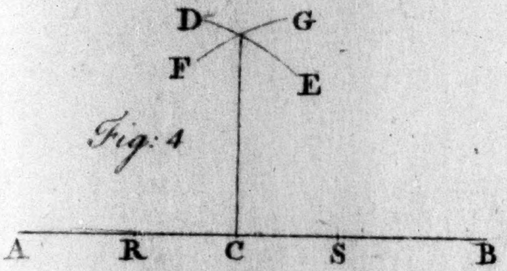

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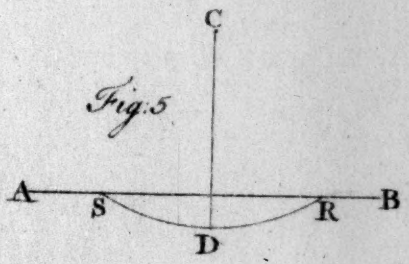
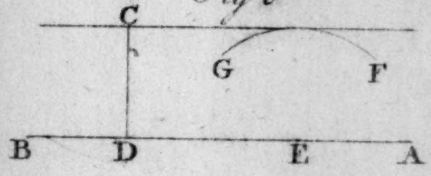


Fig 6



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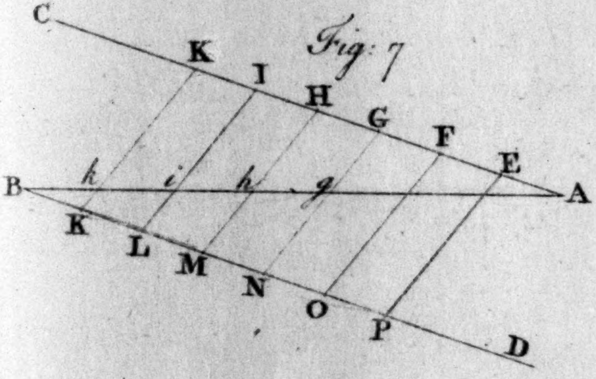

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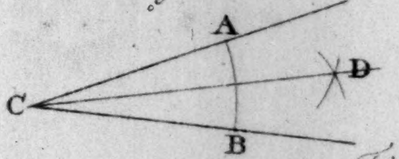

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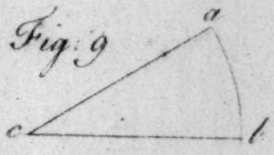

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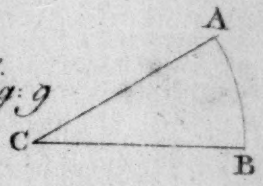

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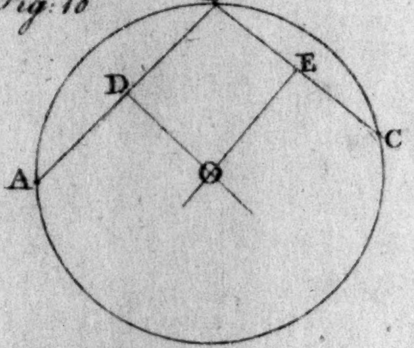

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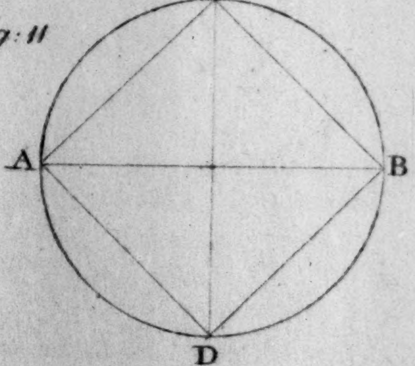

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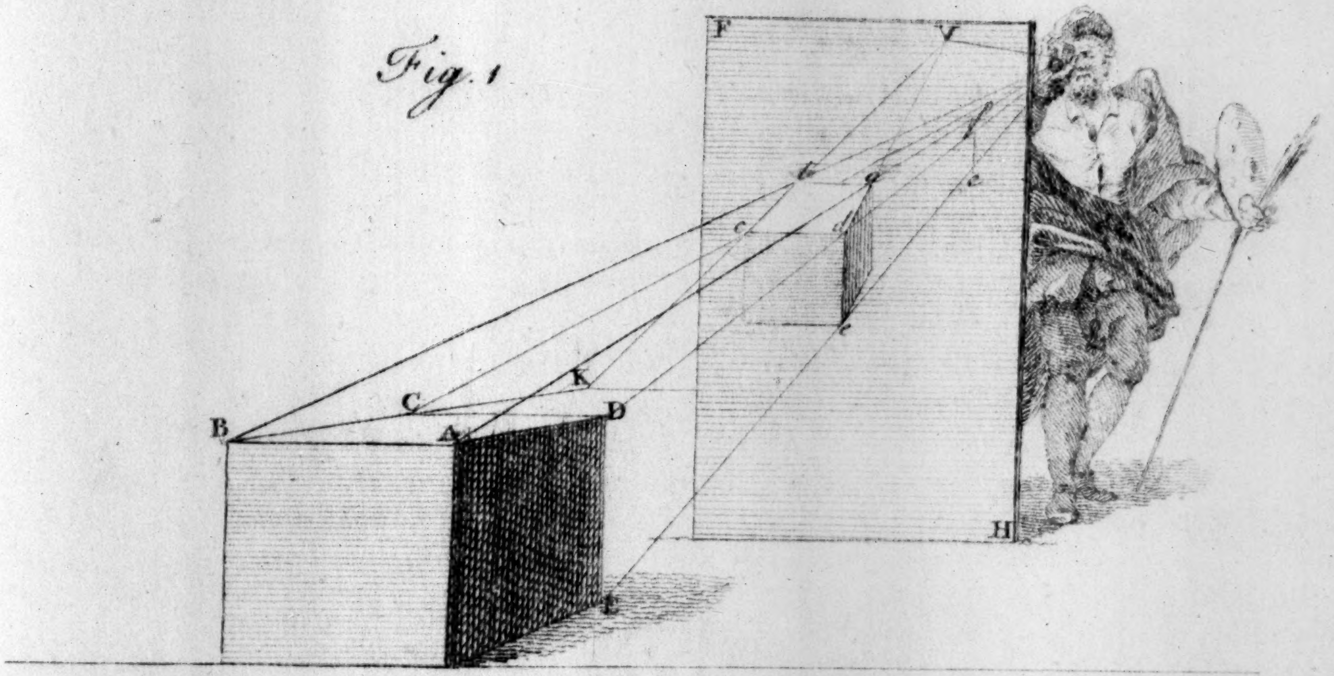
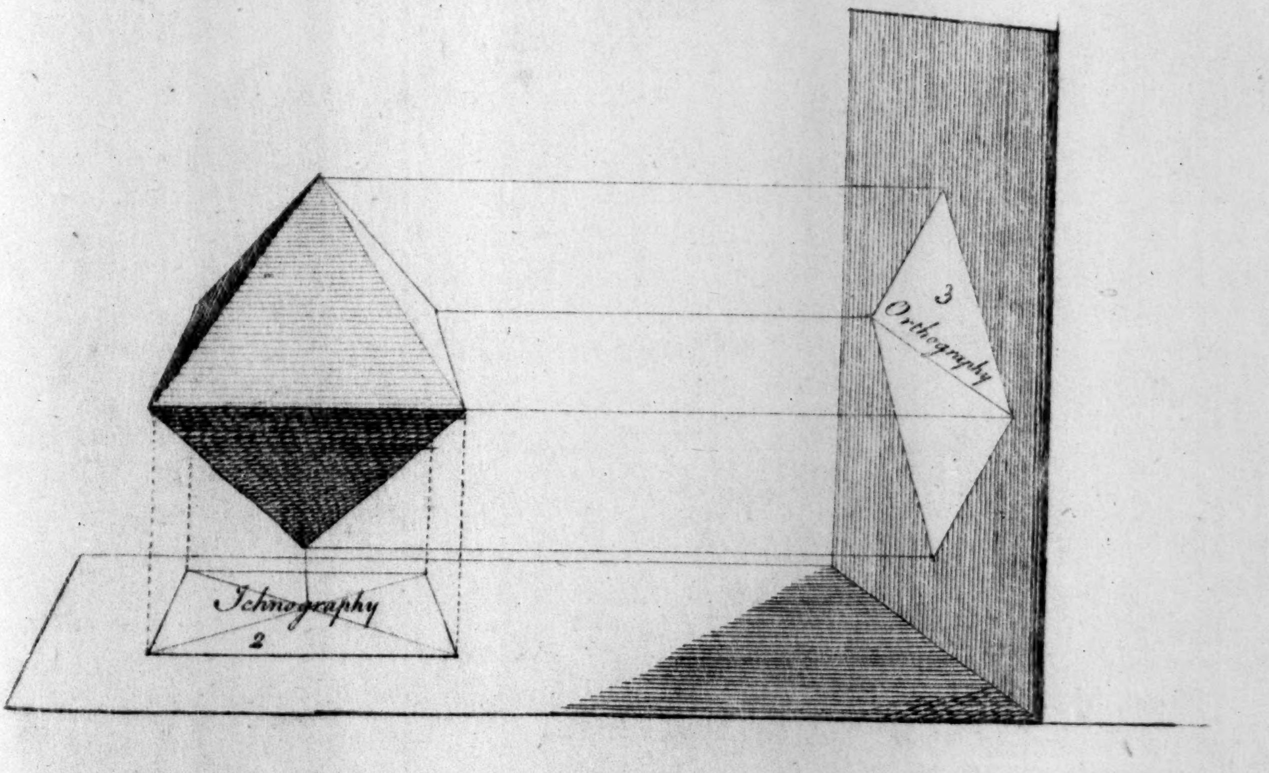
Pl. II

Fig. 1





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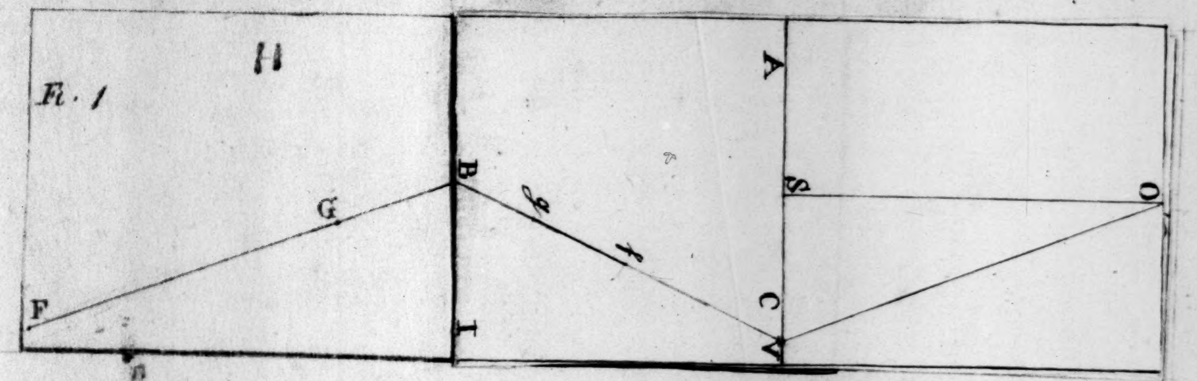
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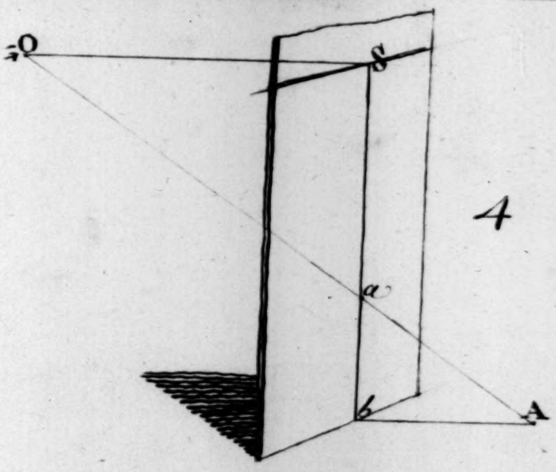
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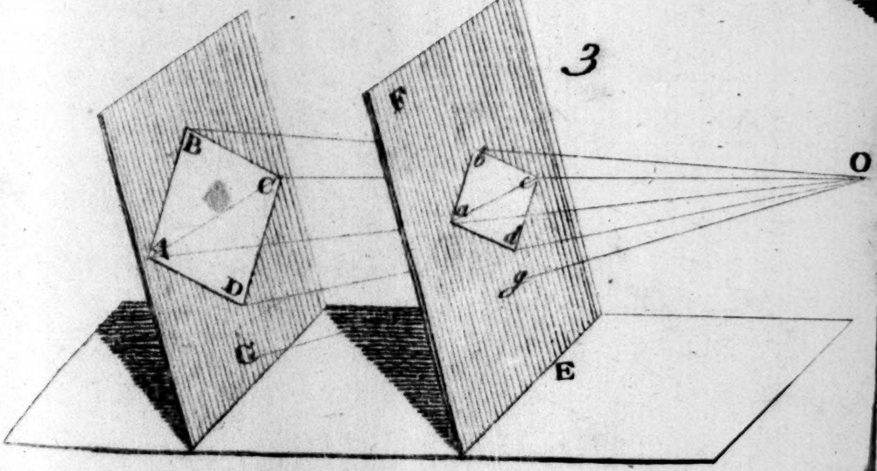
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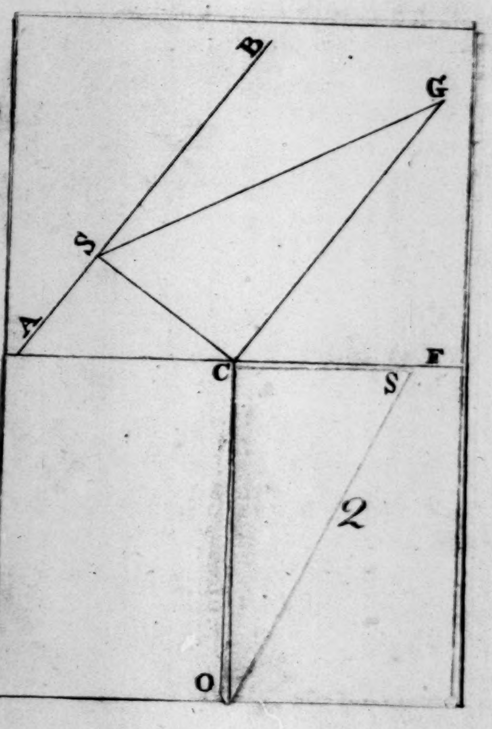
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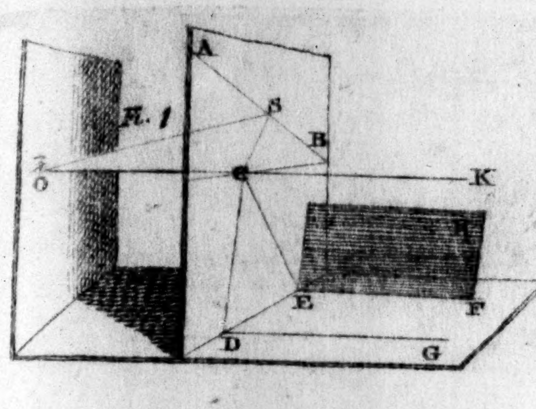
IV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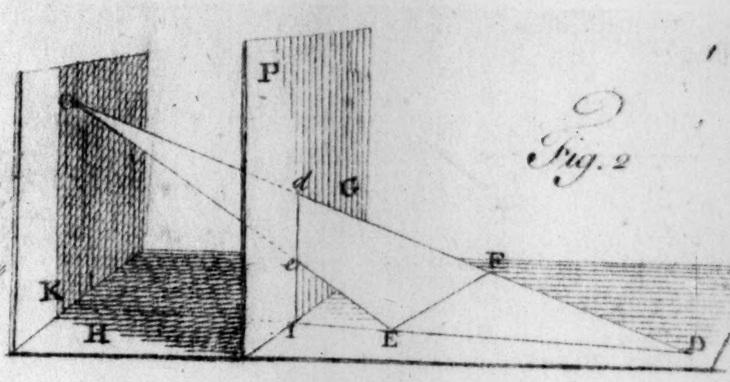
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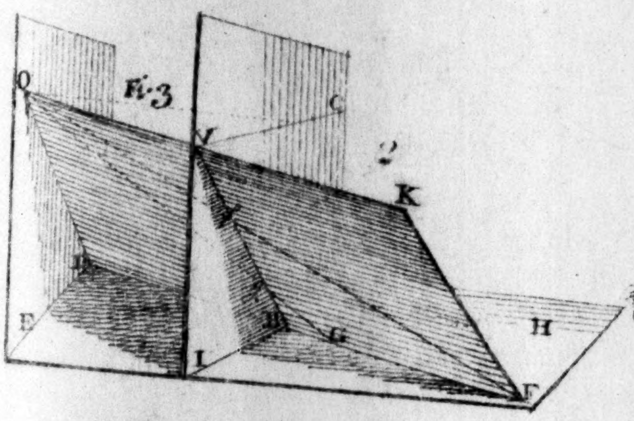
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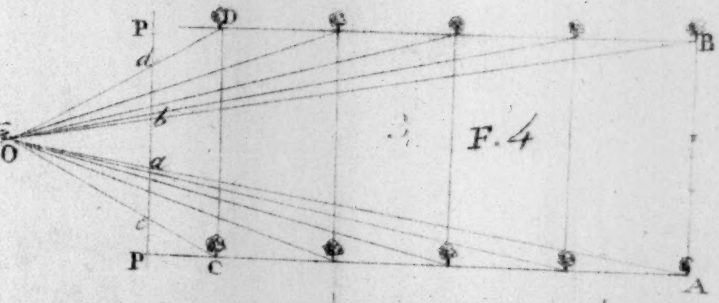
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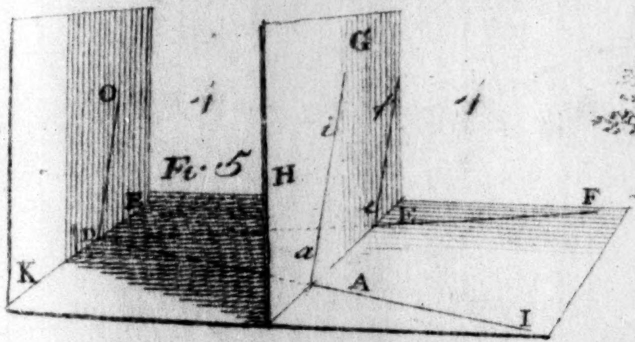
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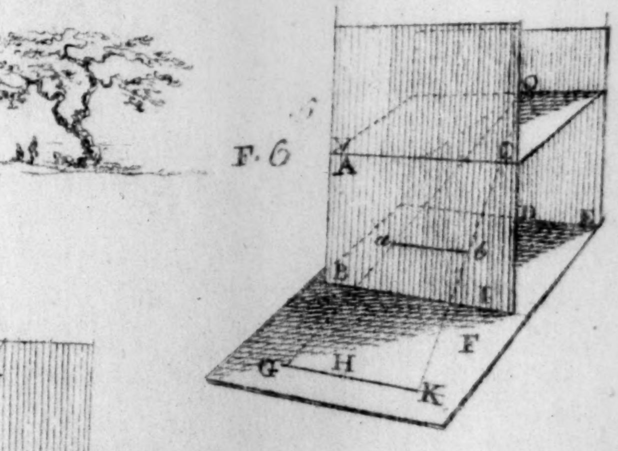
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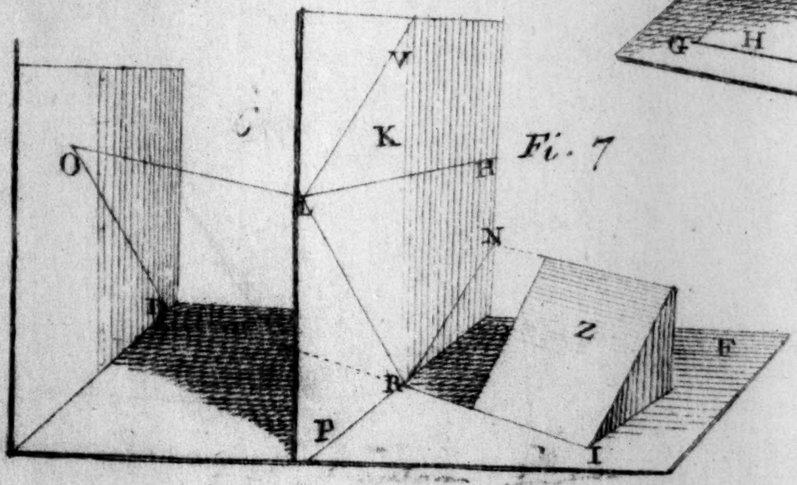
Fi. 7

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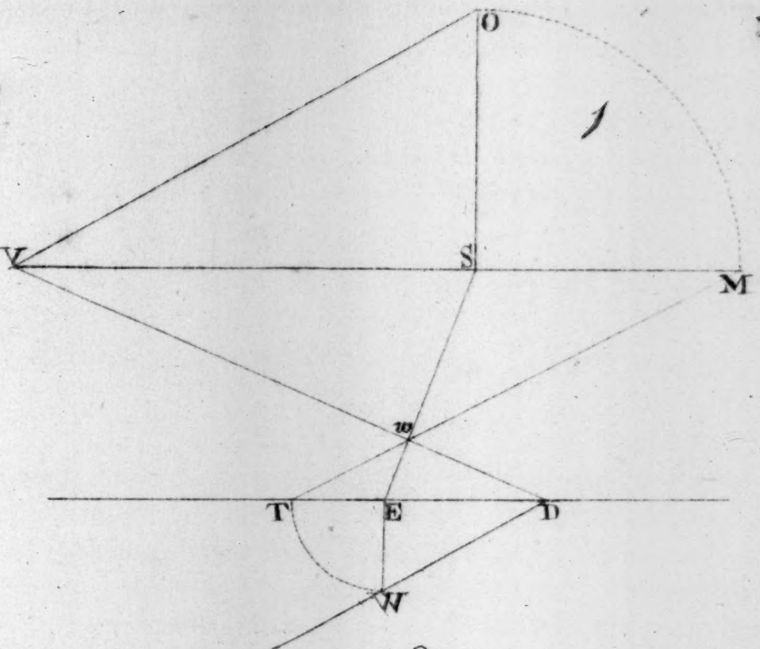
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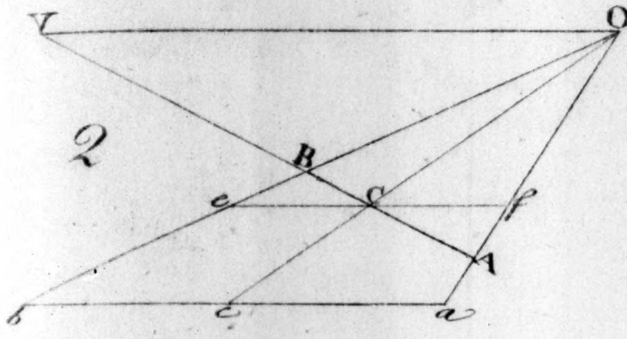
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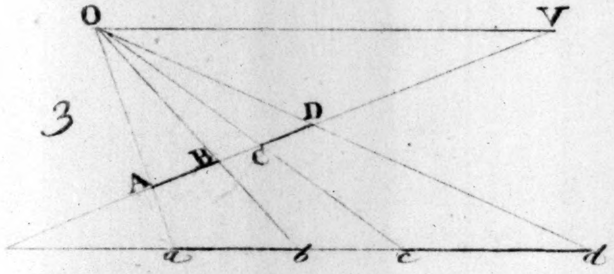
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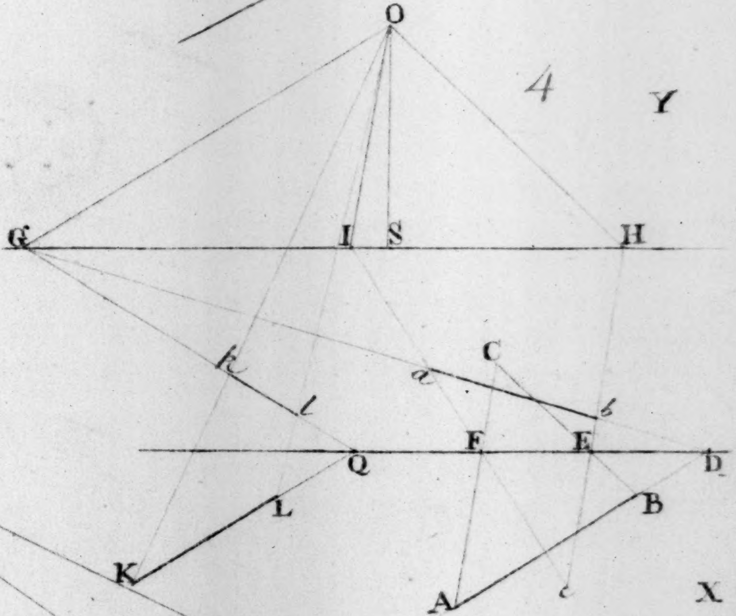
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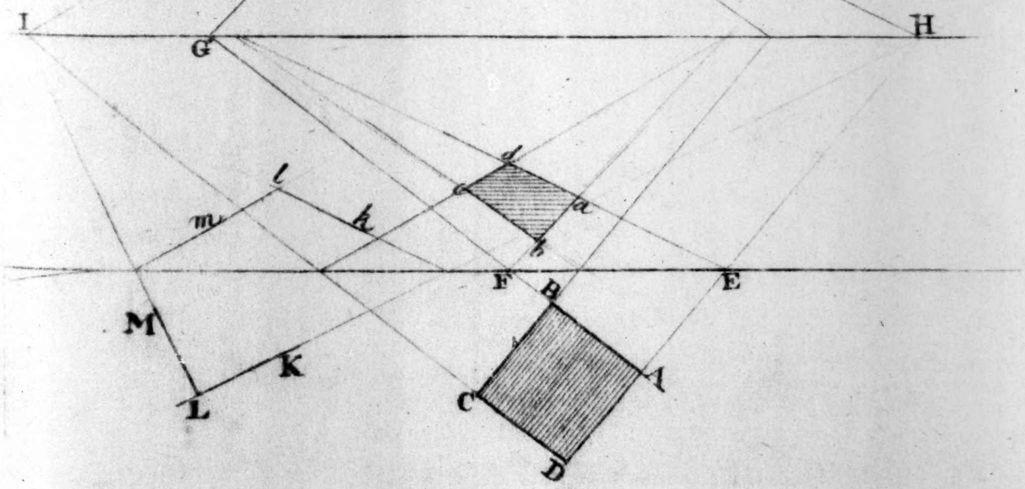
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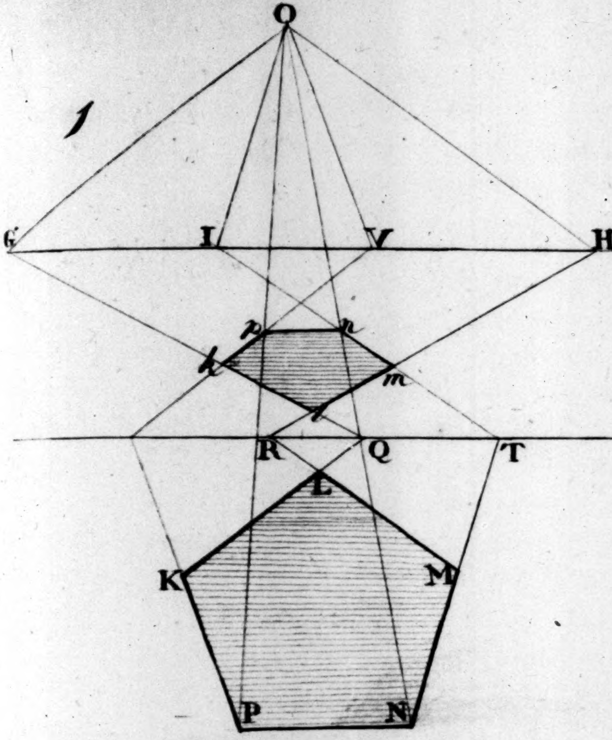

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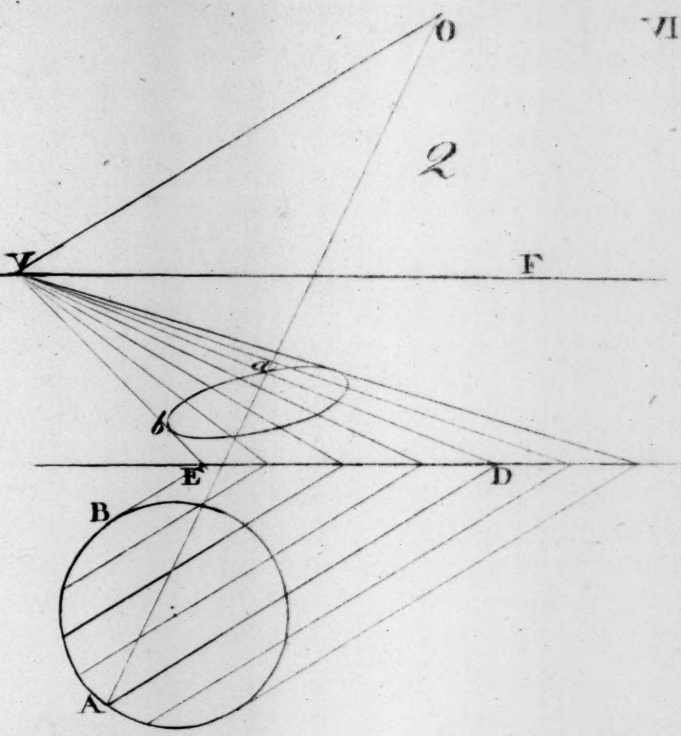

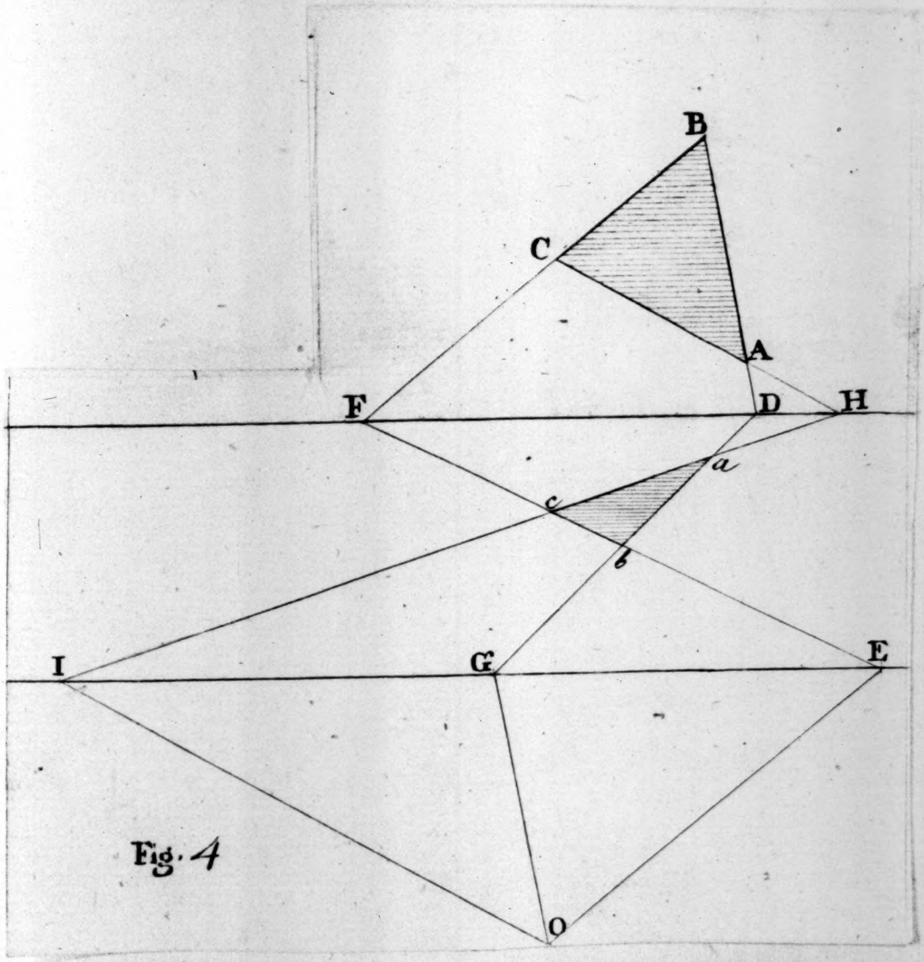

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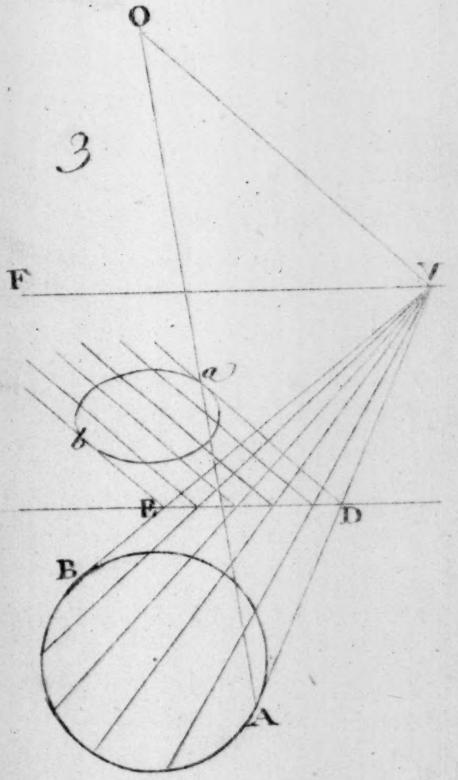

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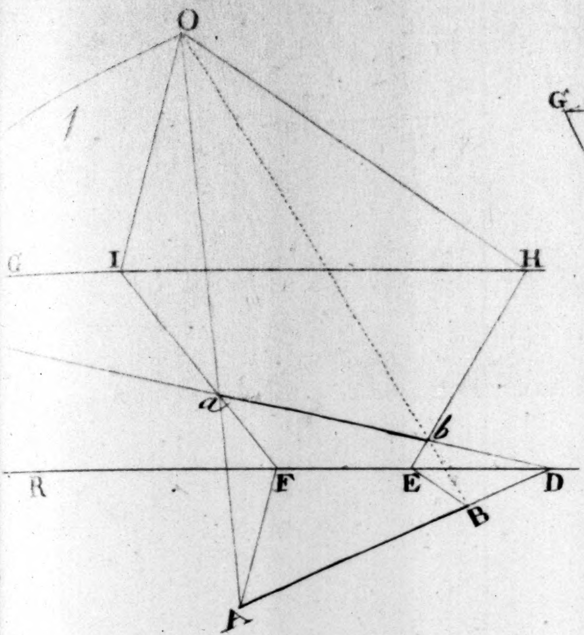
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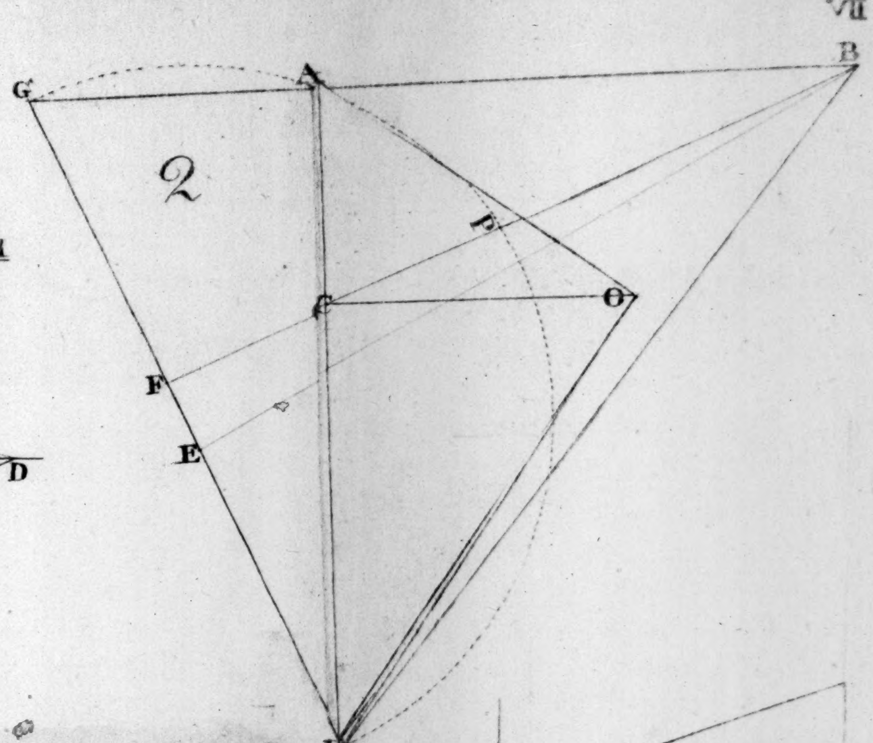
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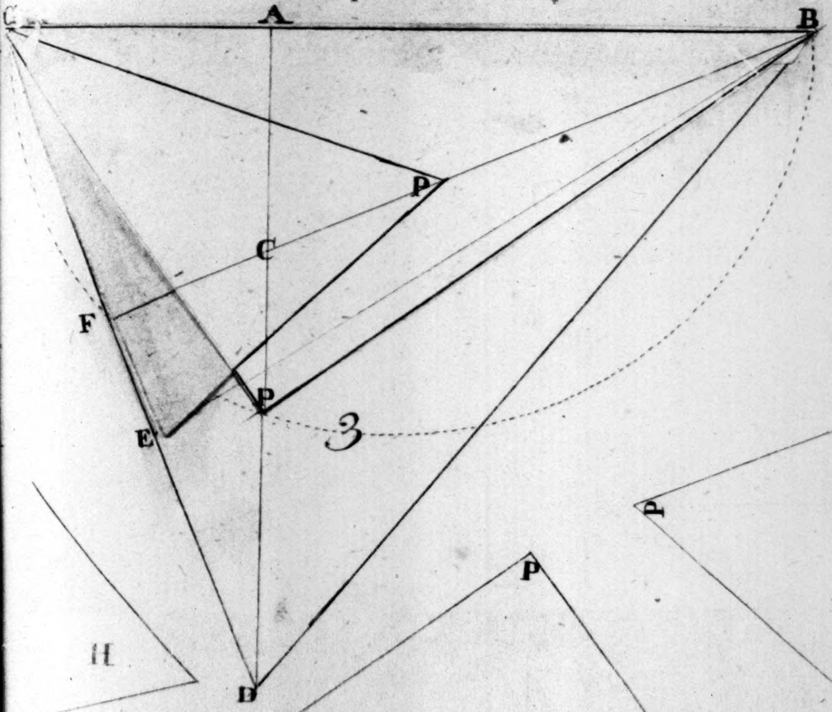
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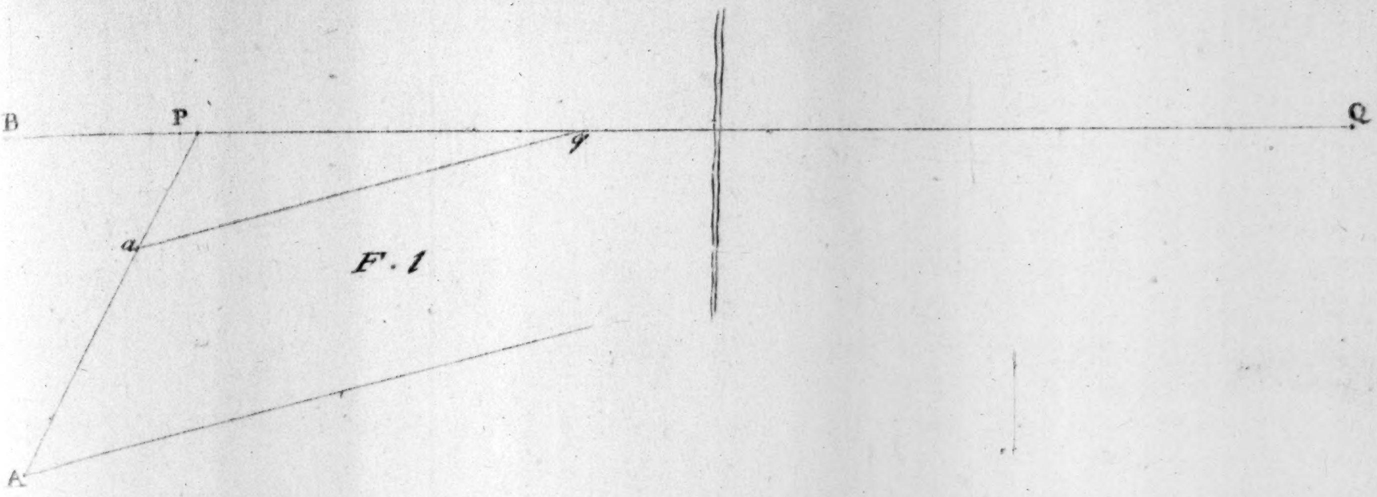

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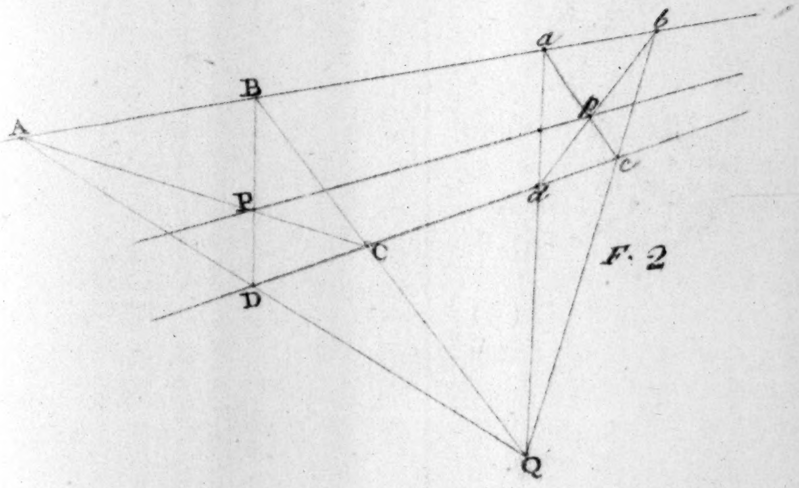

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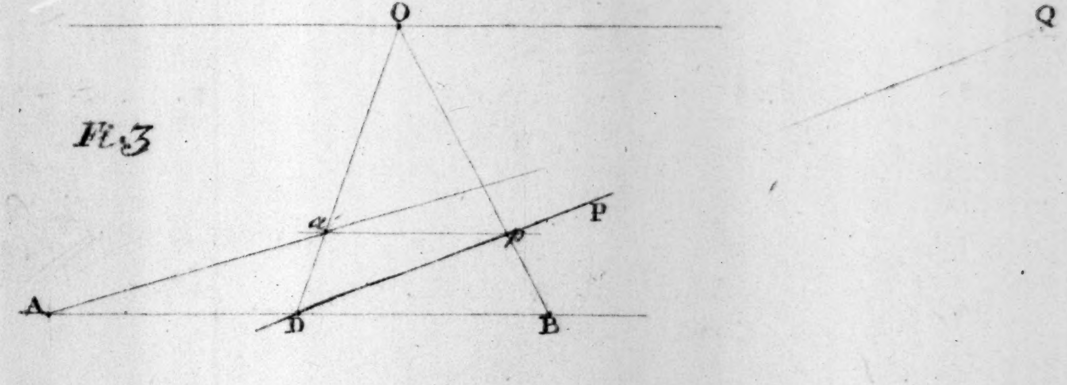

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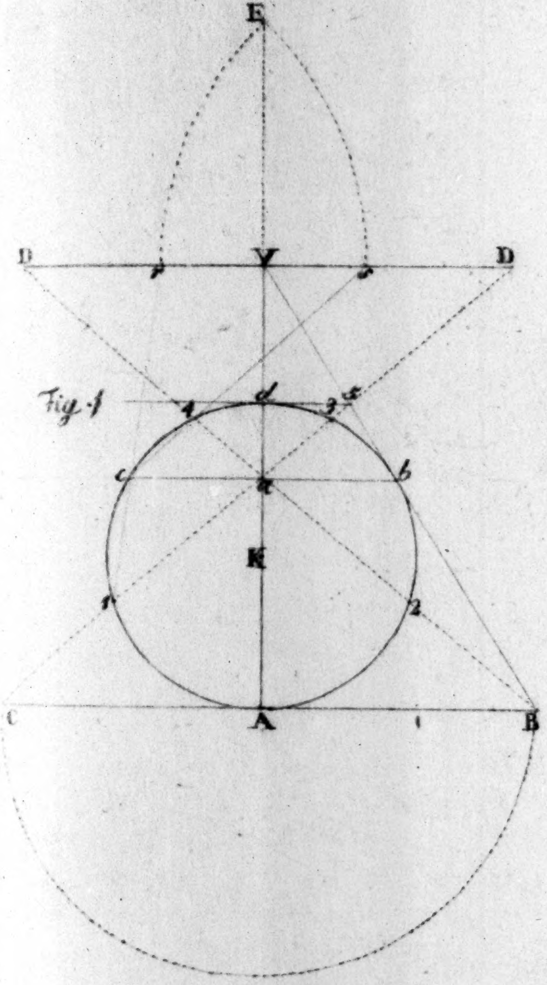
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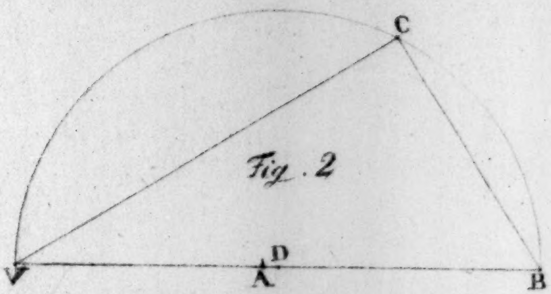
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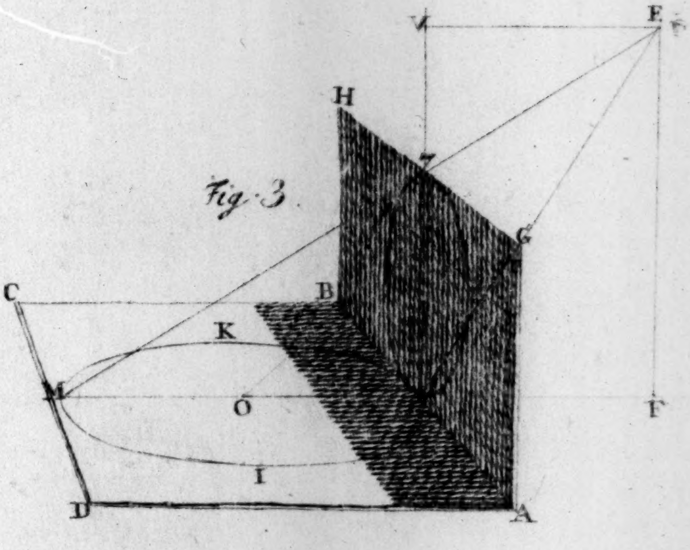
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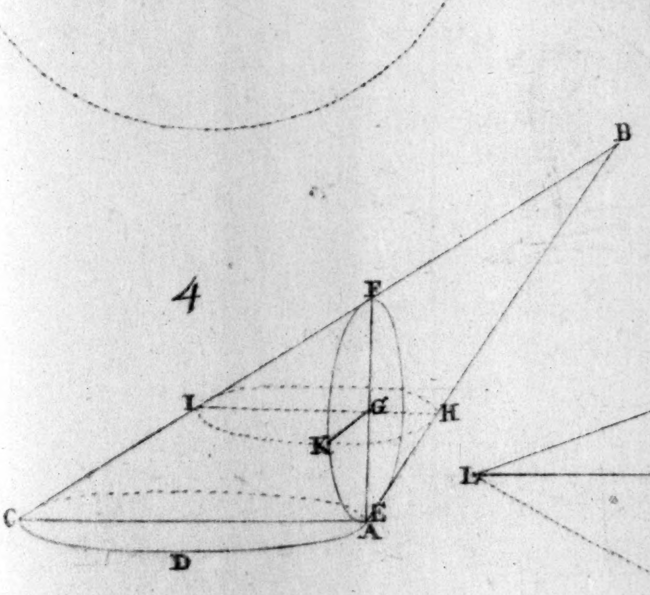
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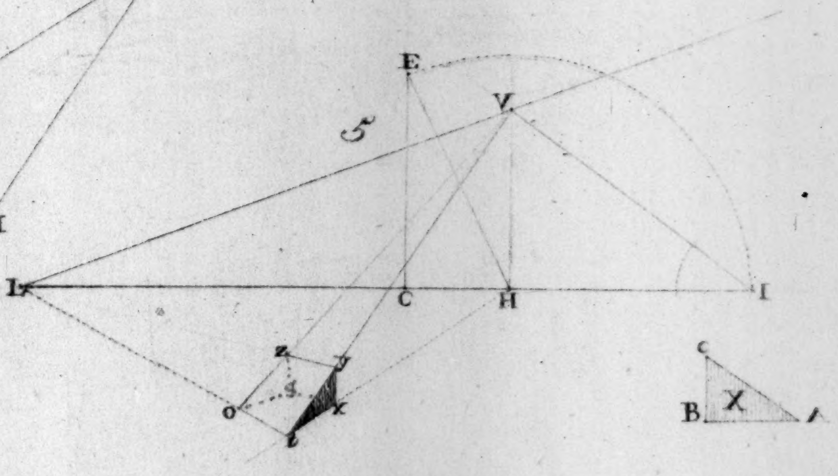
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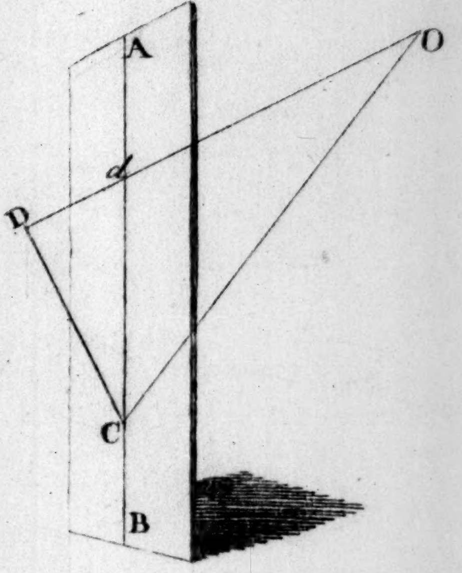

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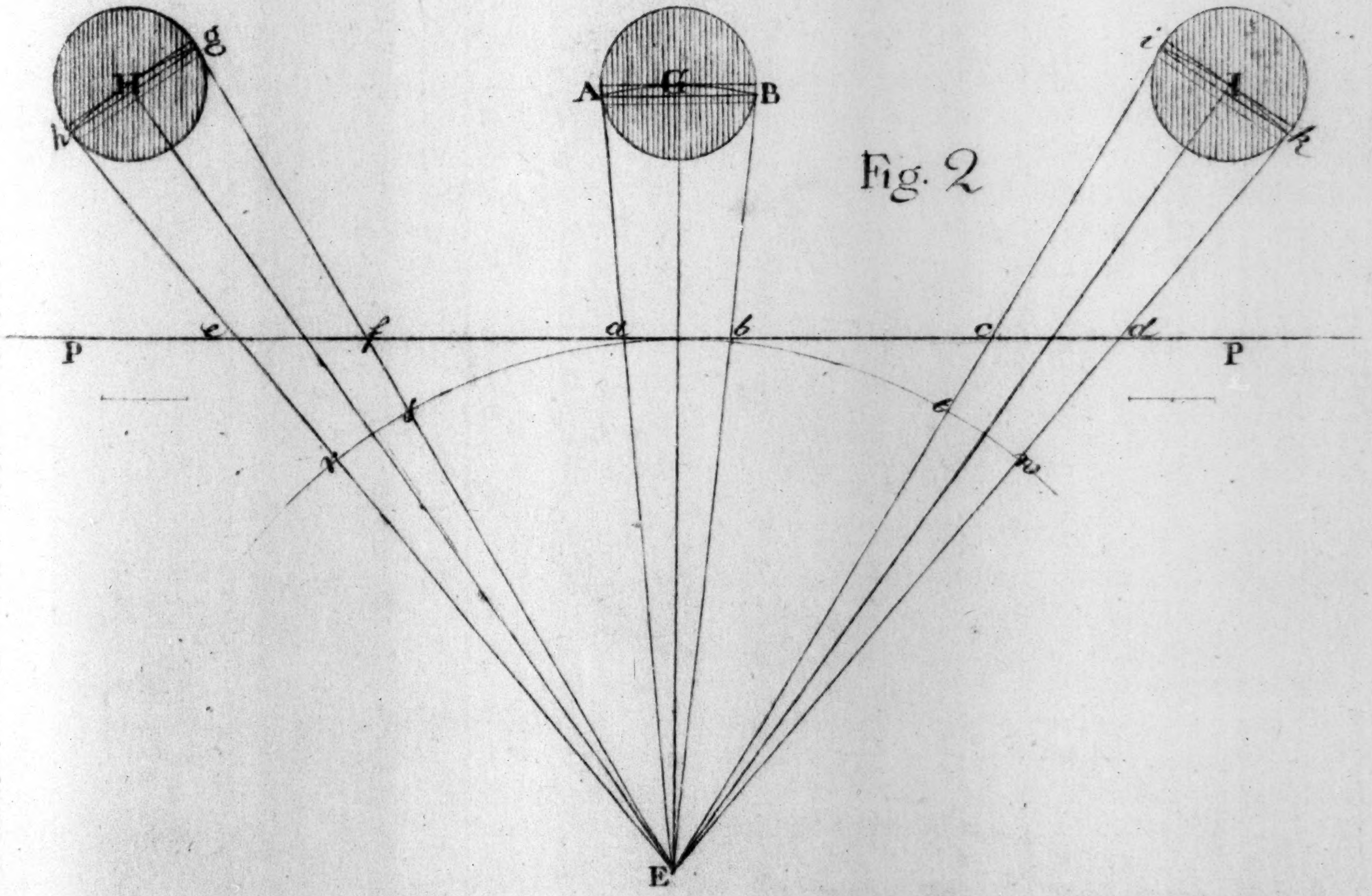

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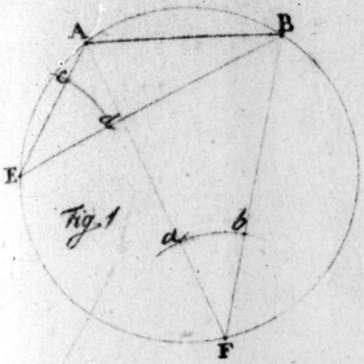

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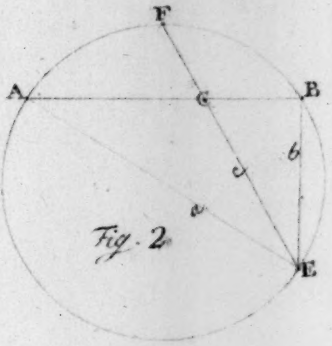

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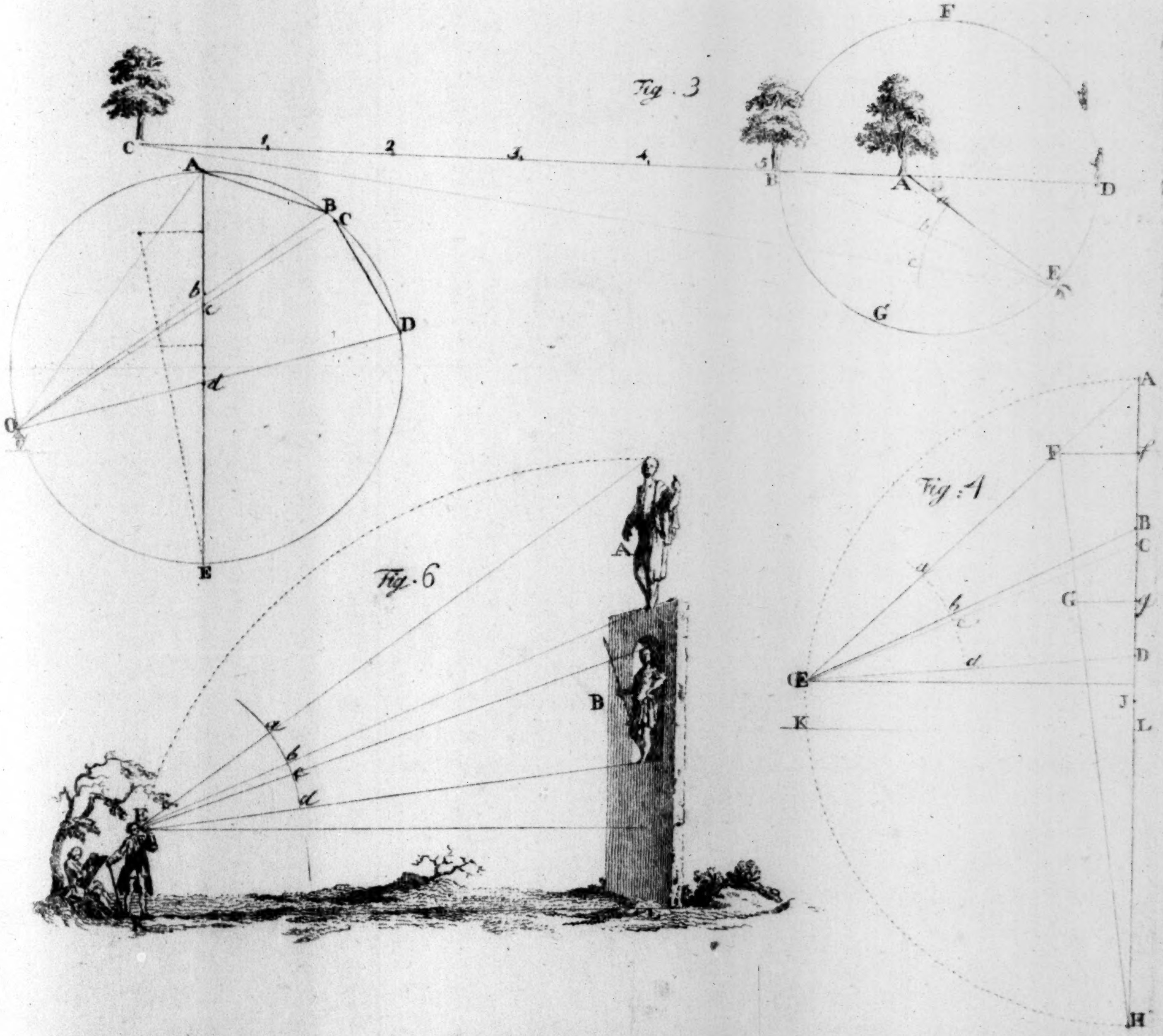

Fig. 3

Fig. 4

Fig. 6

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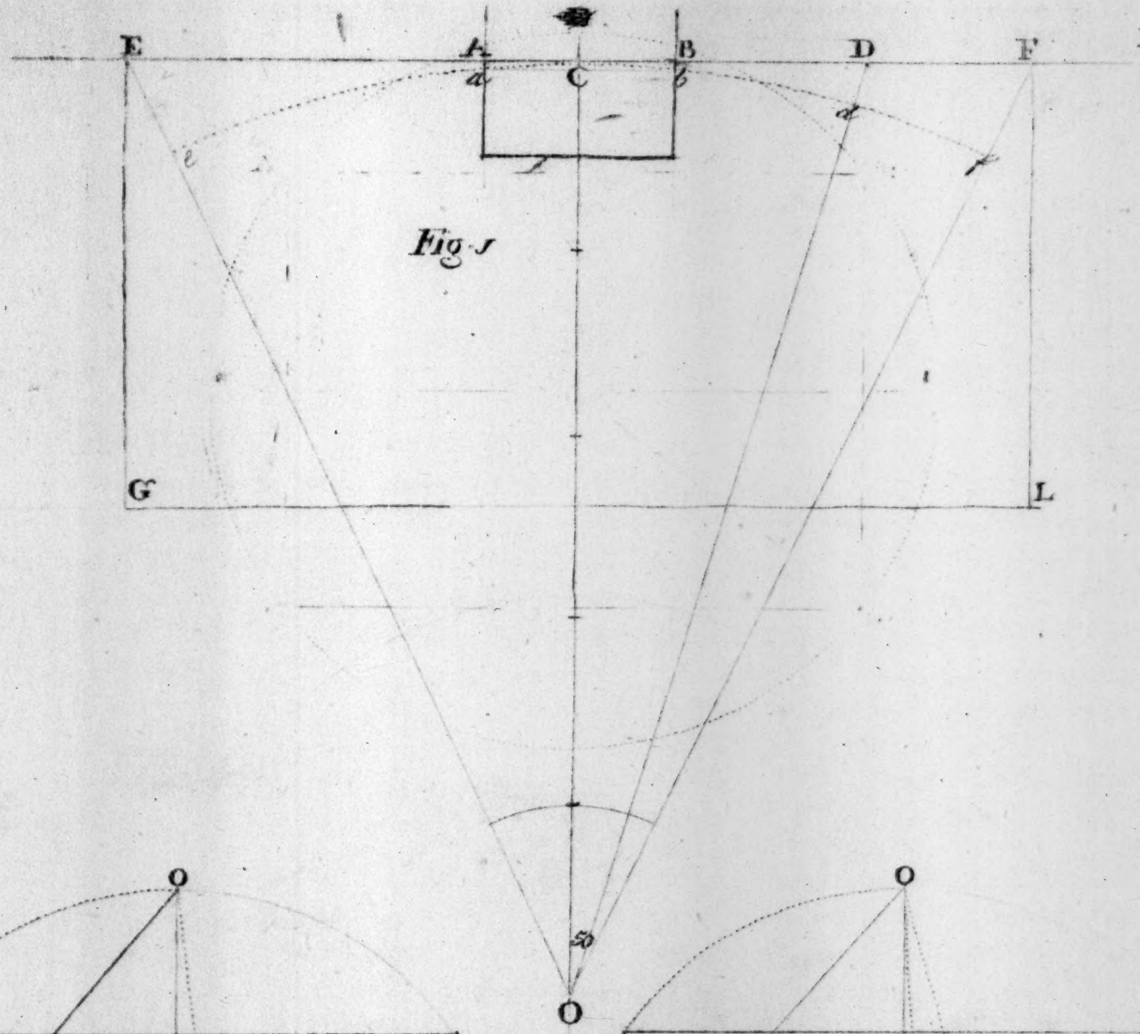

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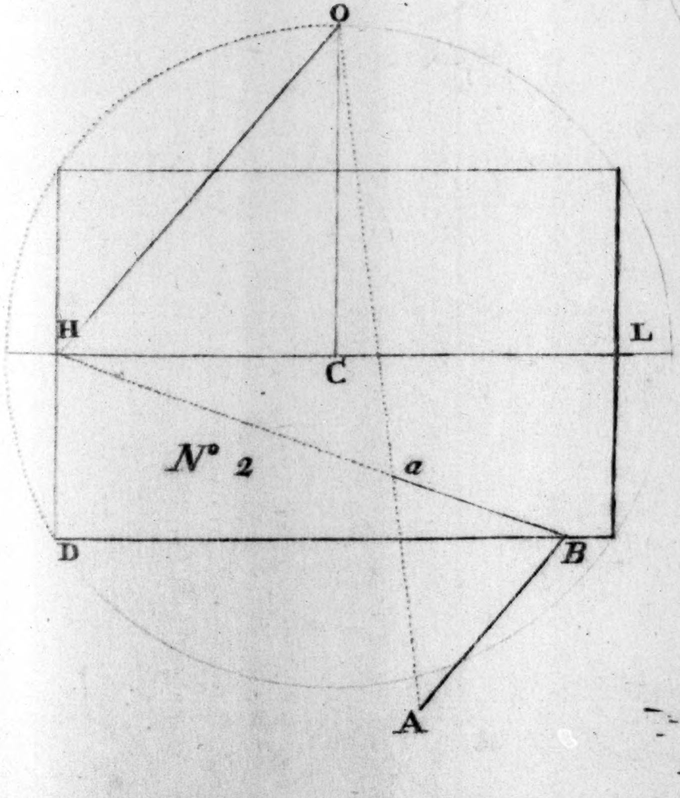

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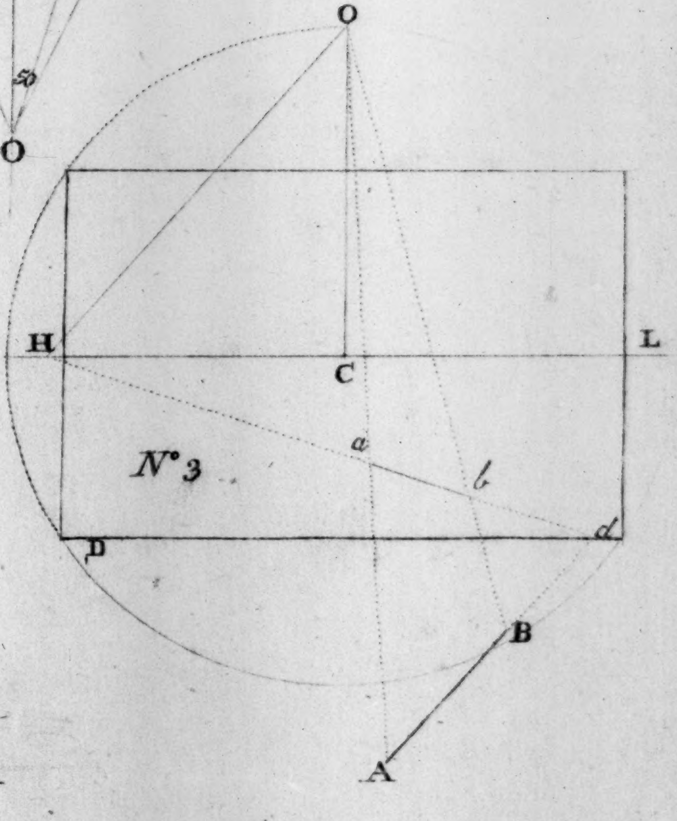

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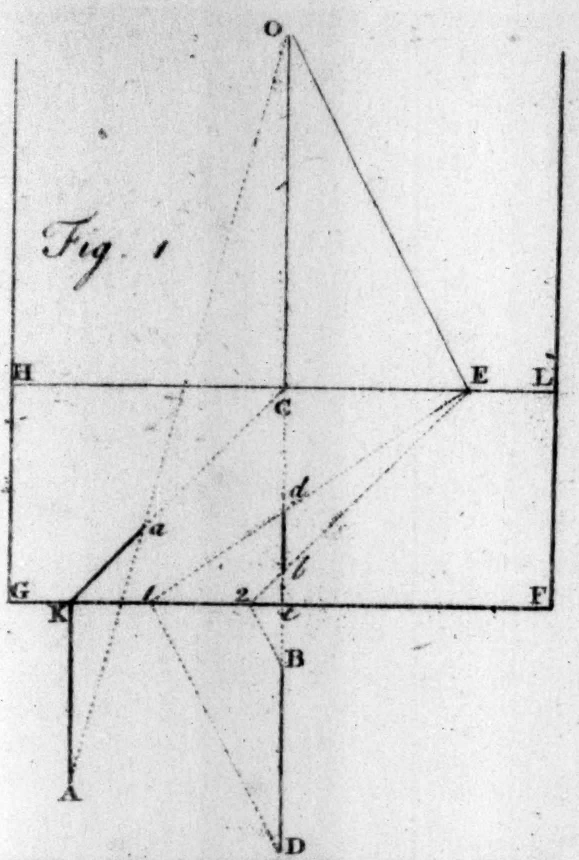
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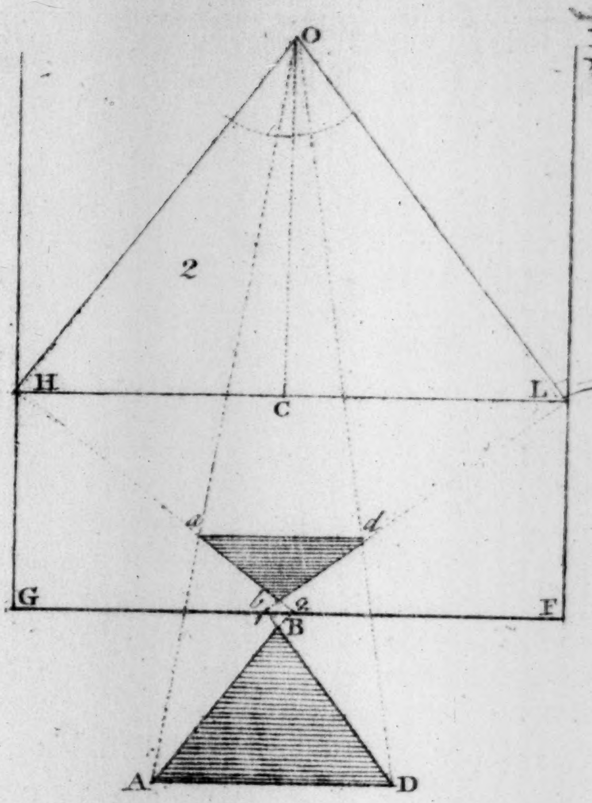
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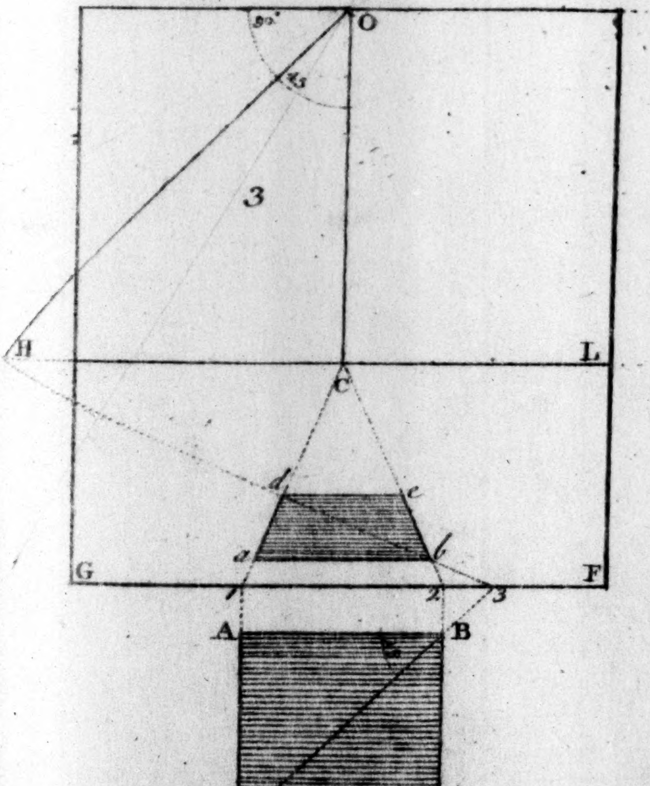
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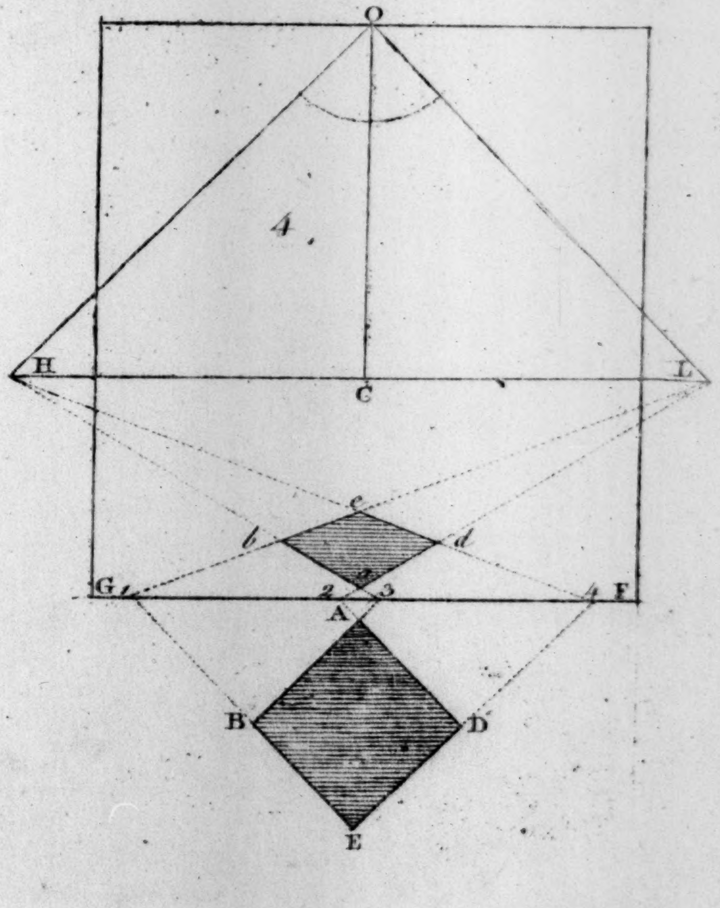
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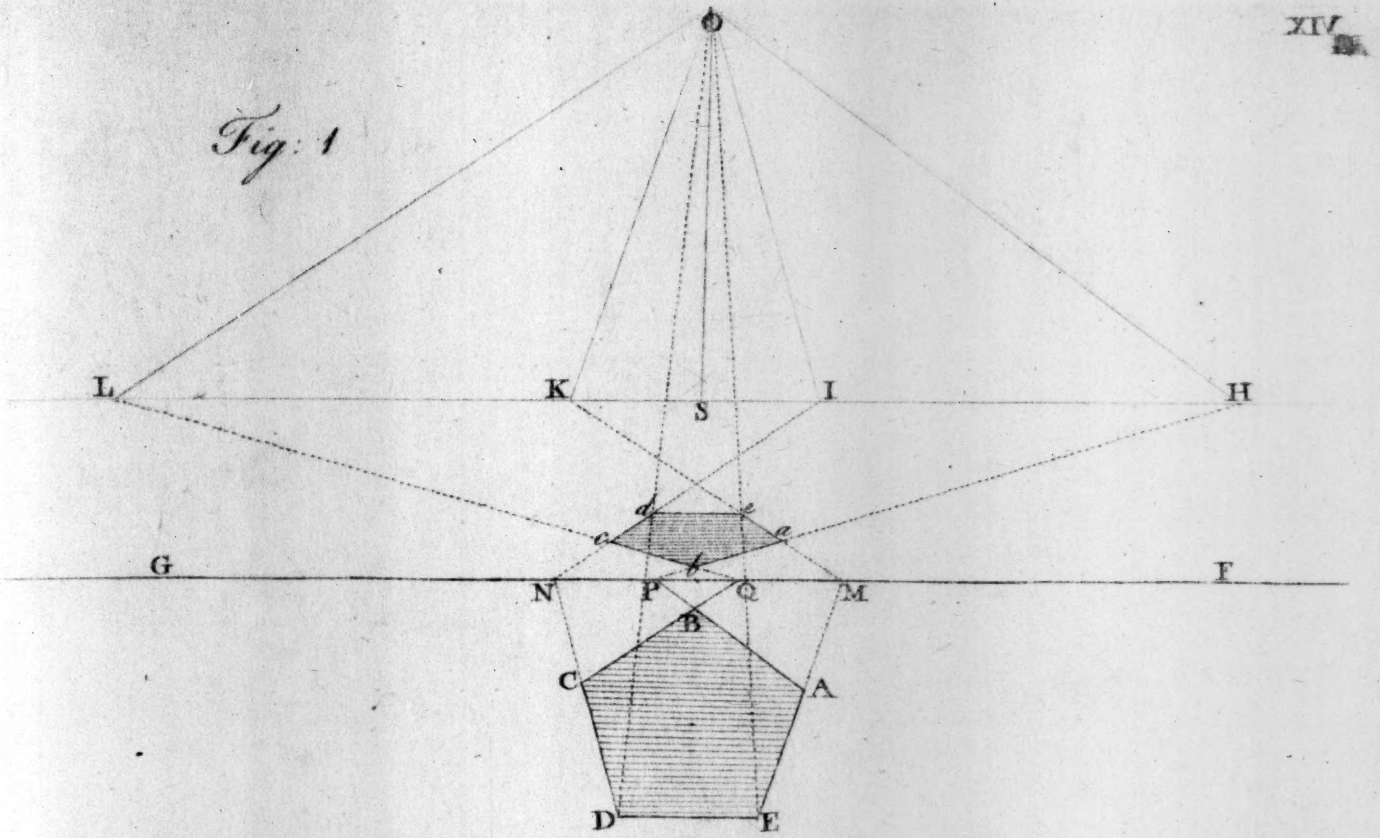
Fig: 1



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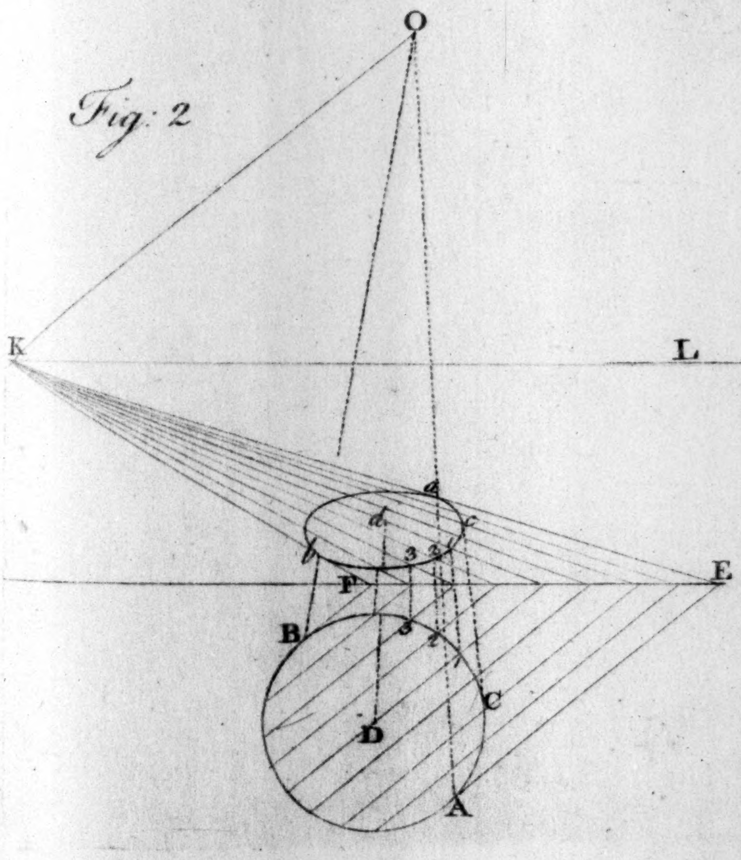

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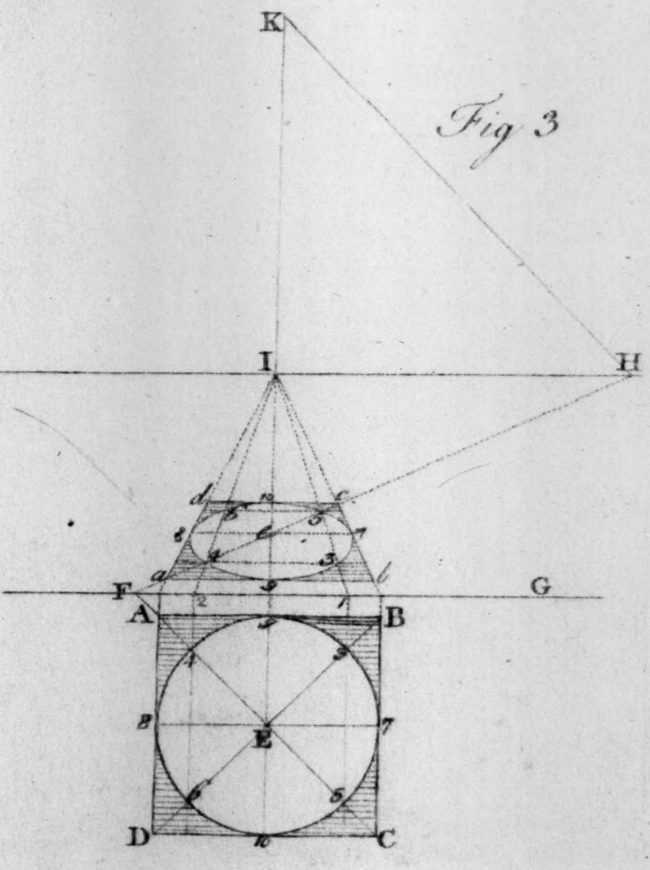

BRITISH
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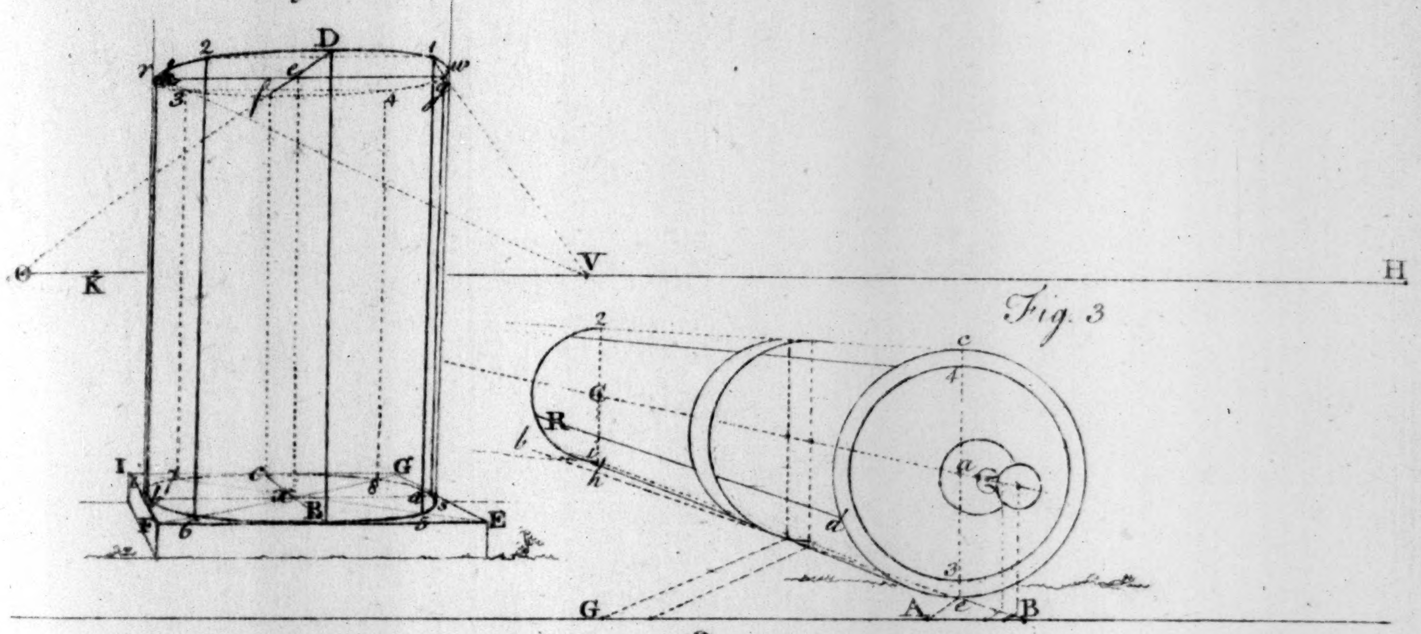
MUSEUM

Pl. XV

Fig. 1

Fig. 3



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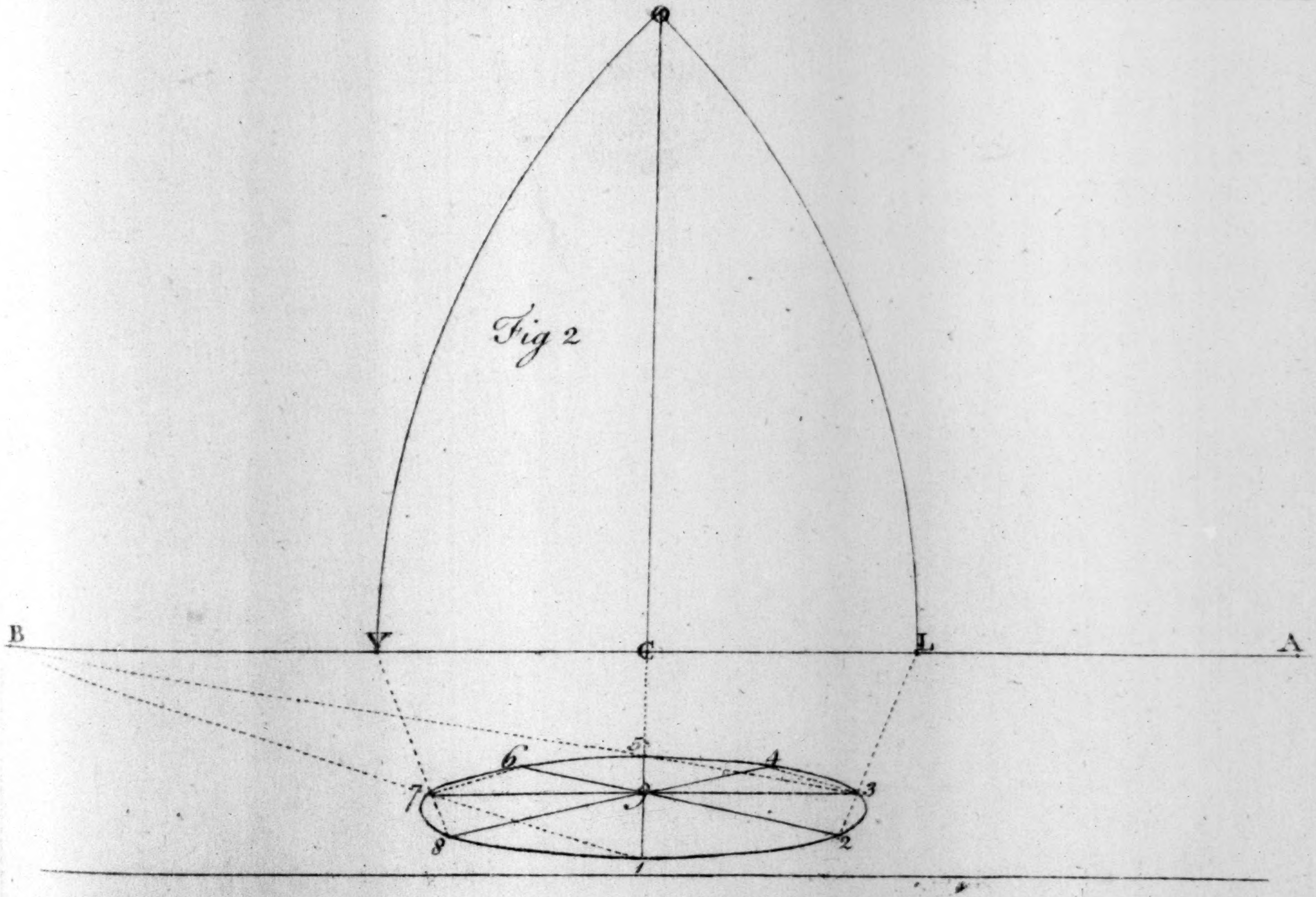

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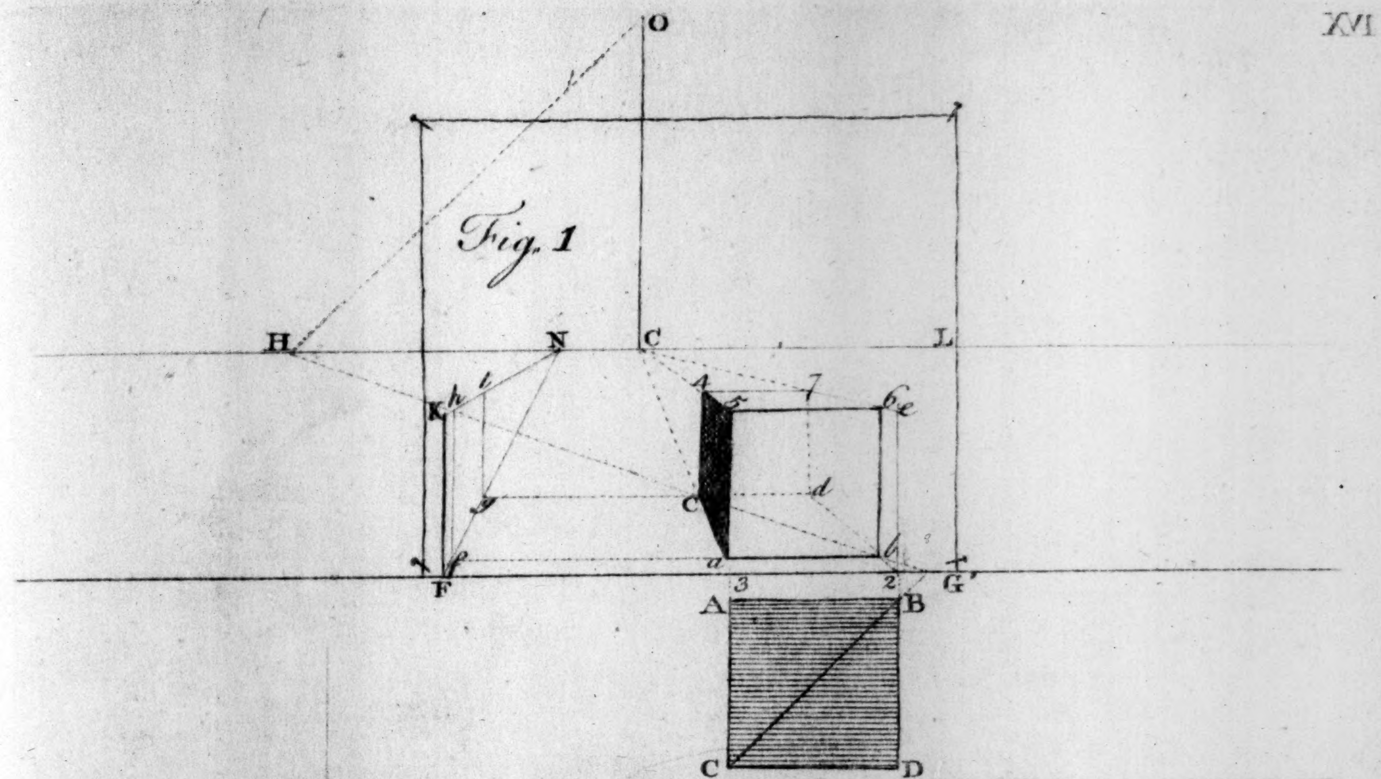
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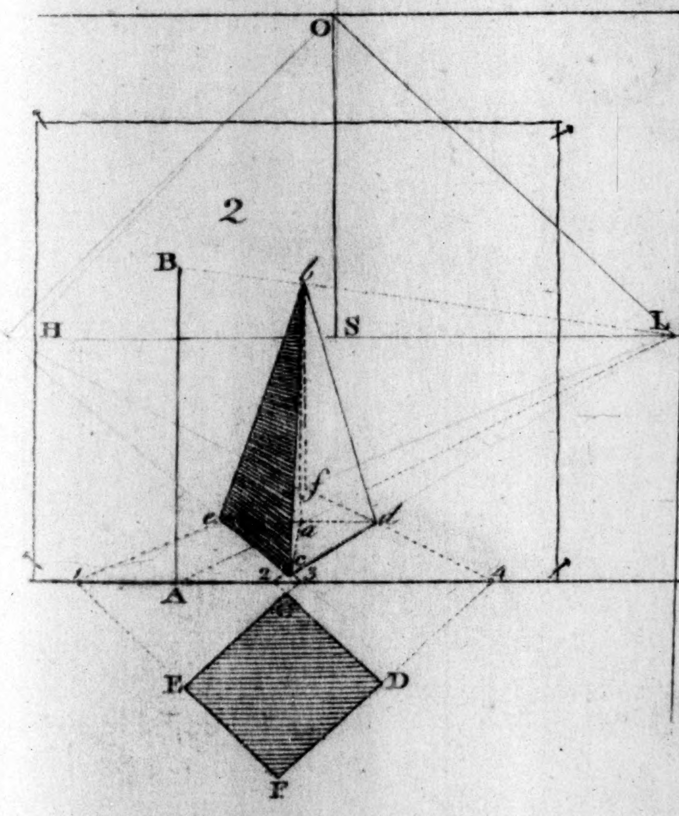
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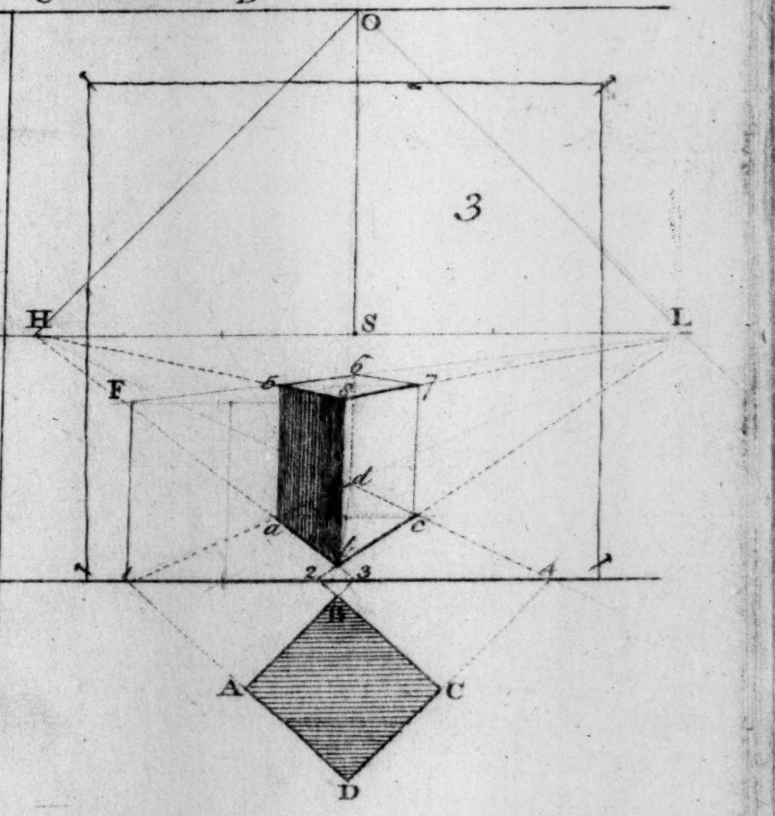
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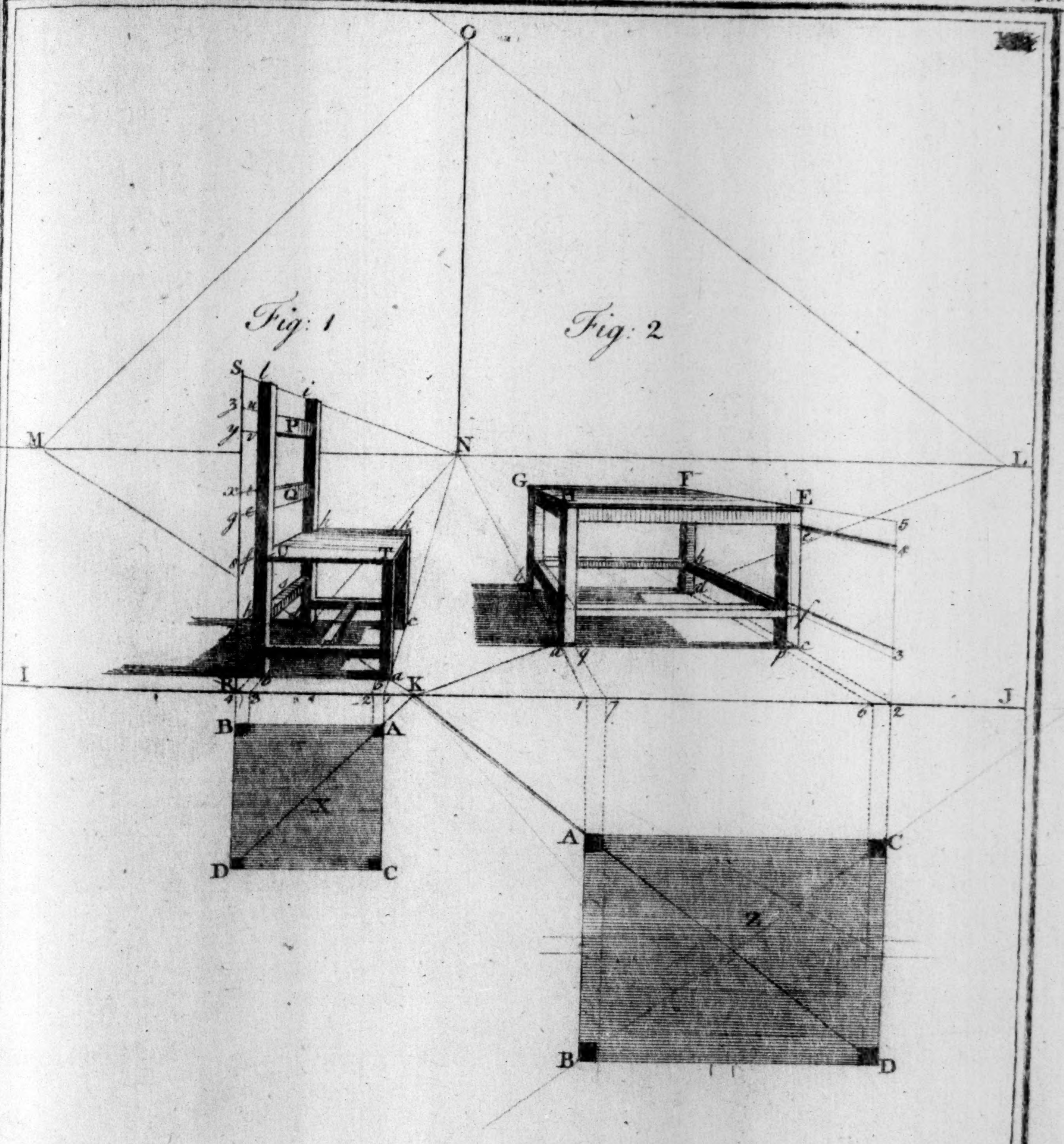
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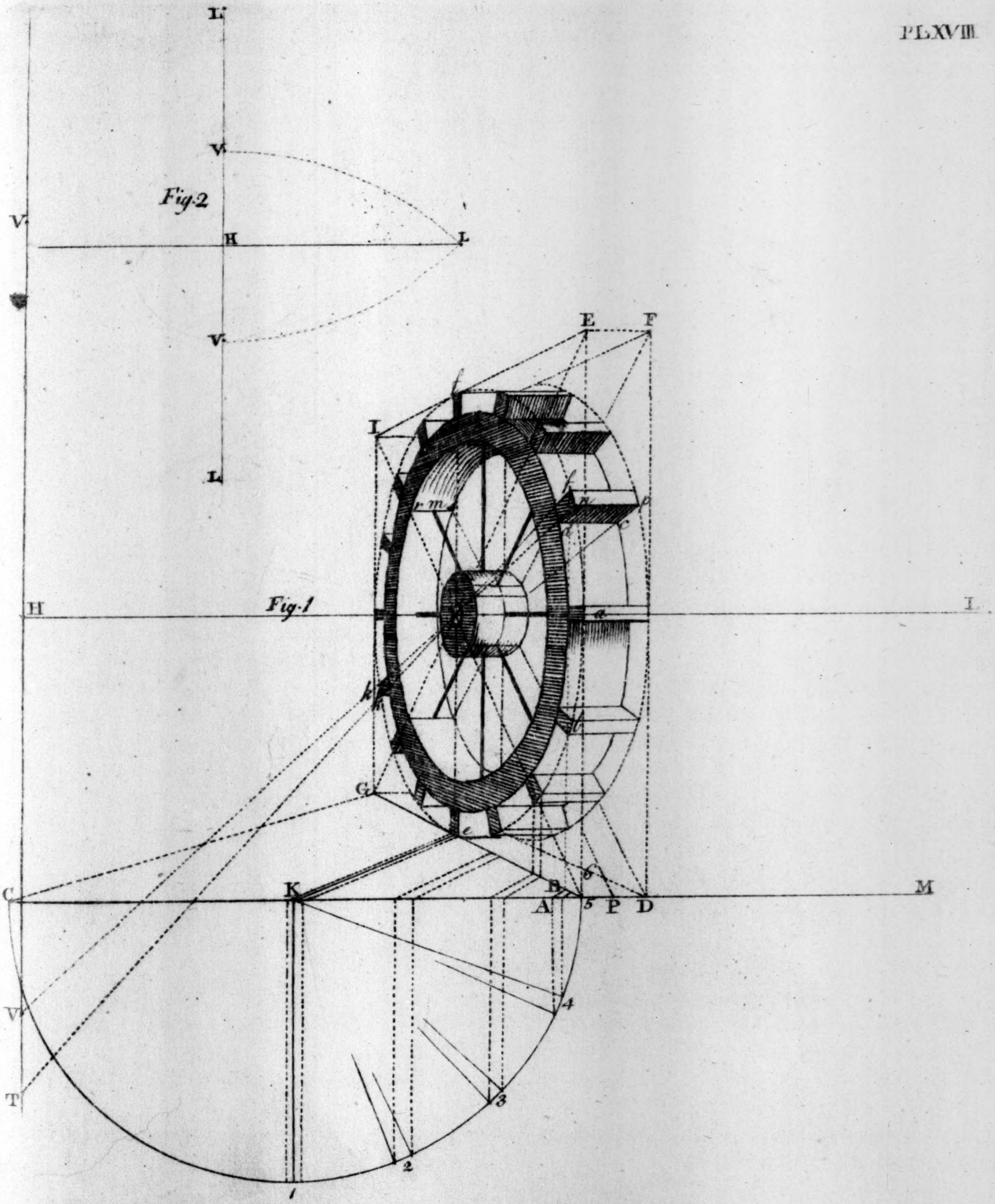
PL.XVIII



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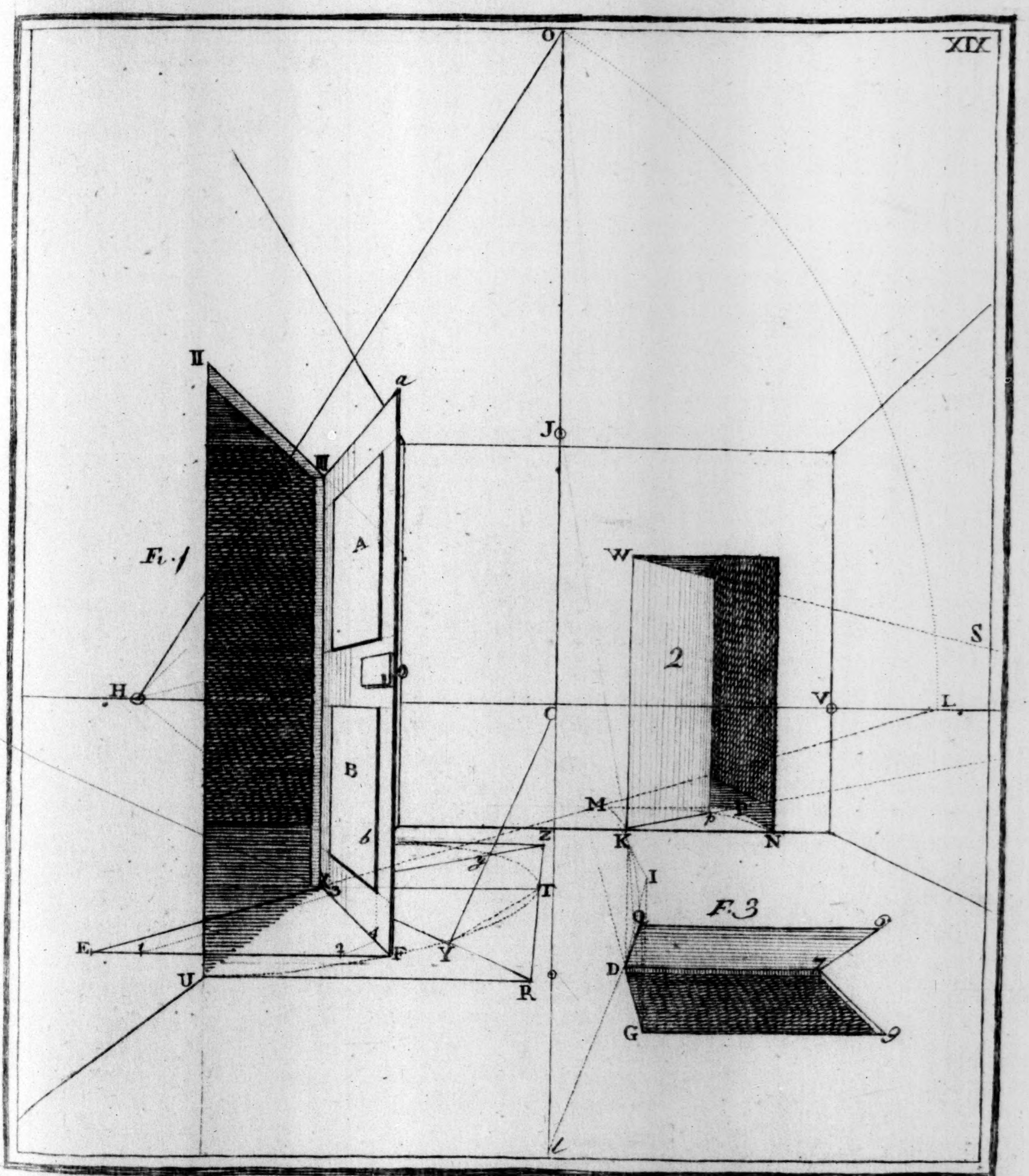
XIX



BRITISH 3 MY 76 MUSEUM

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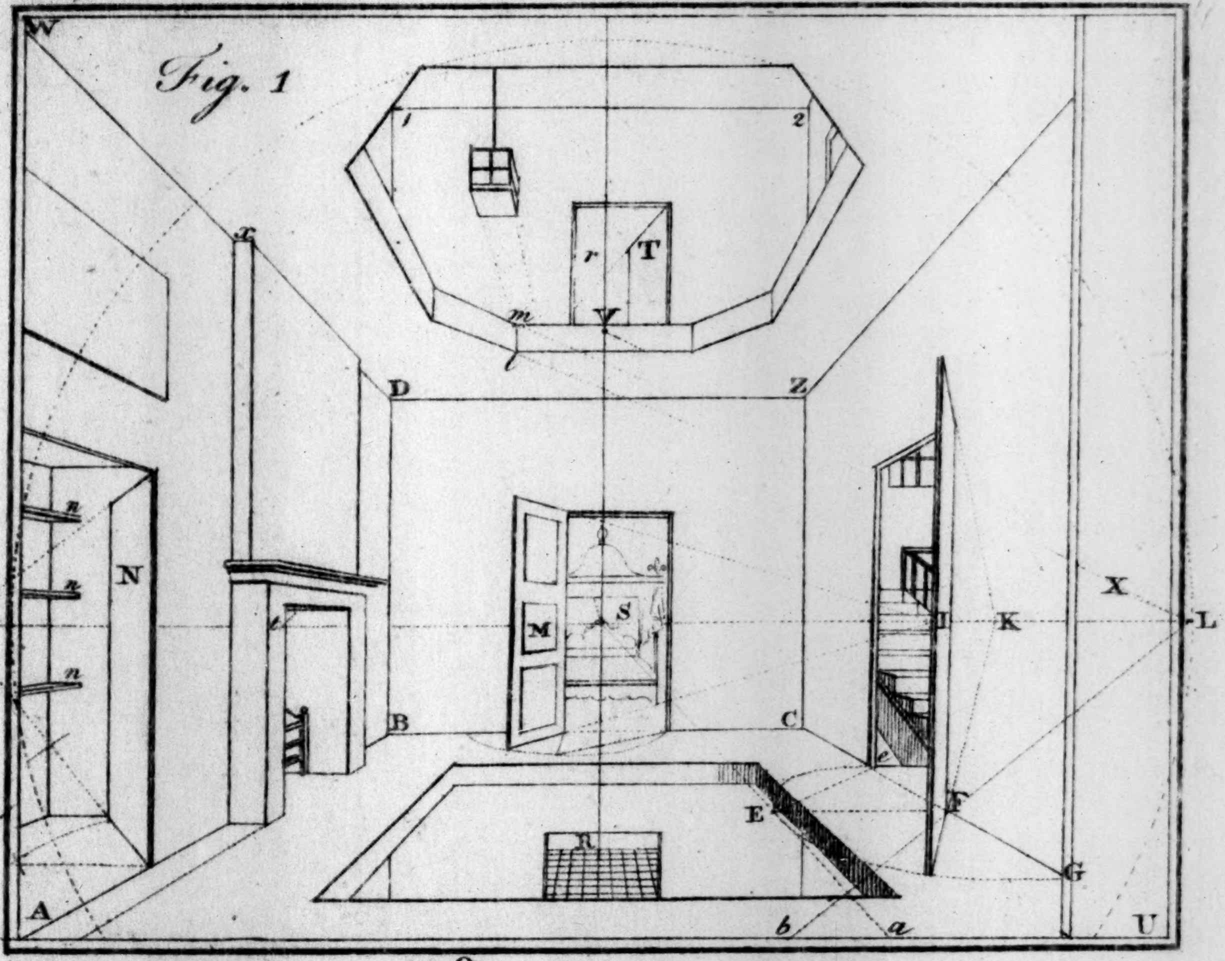
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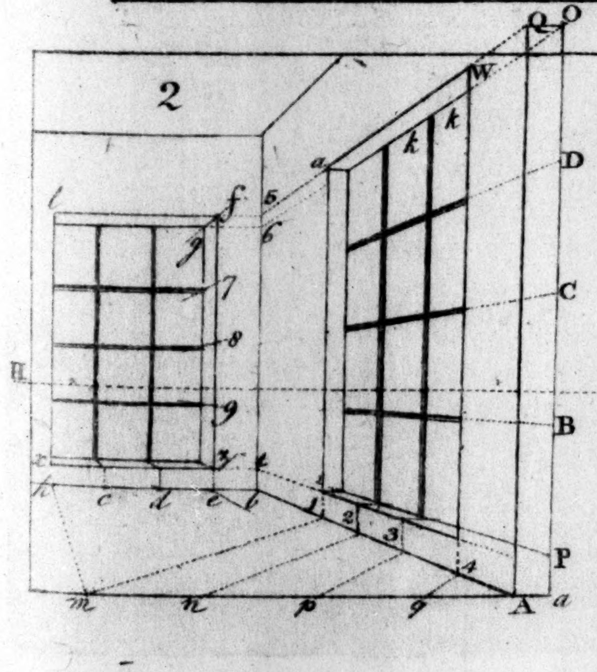
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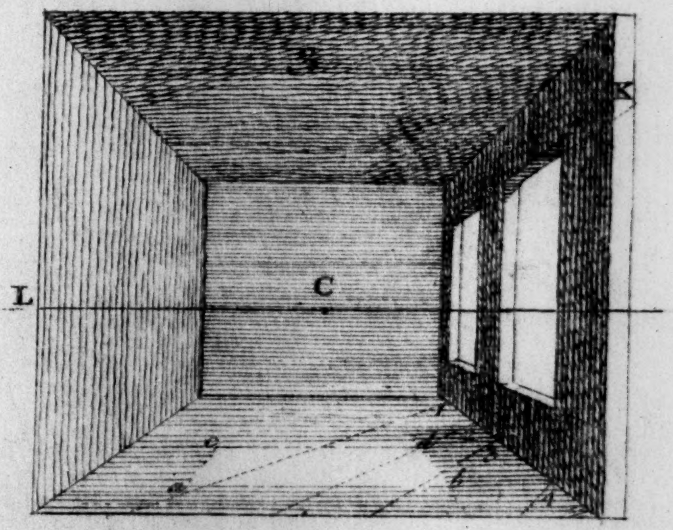
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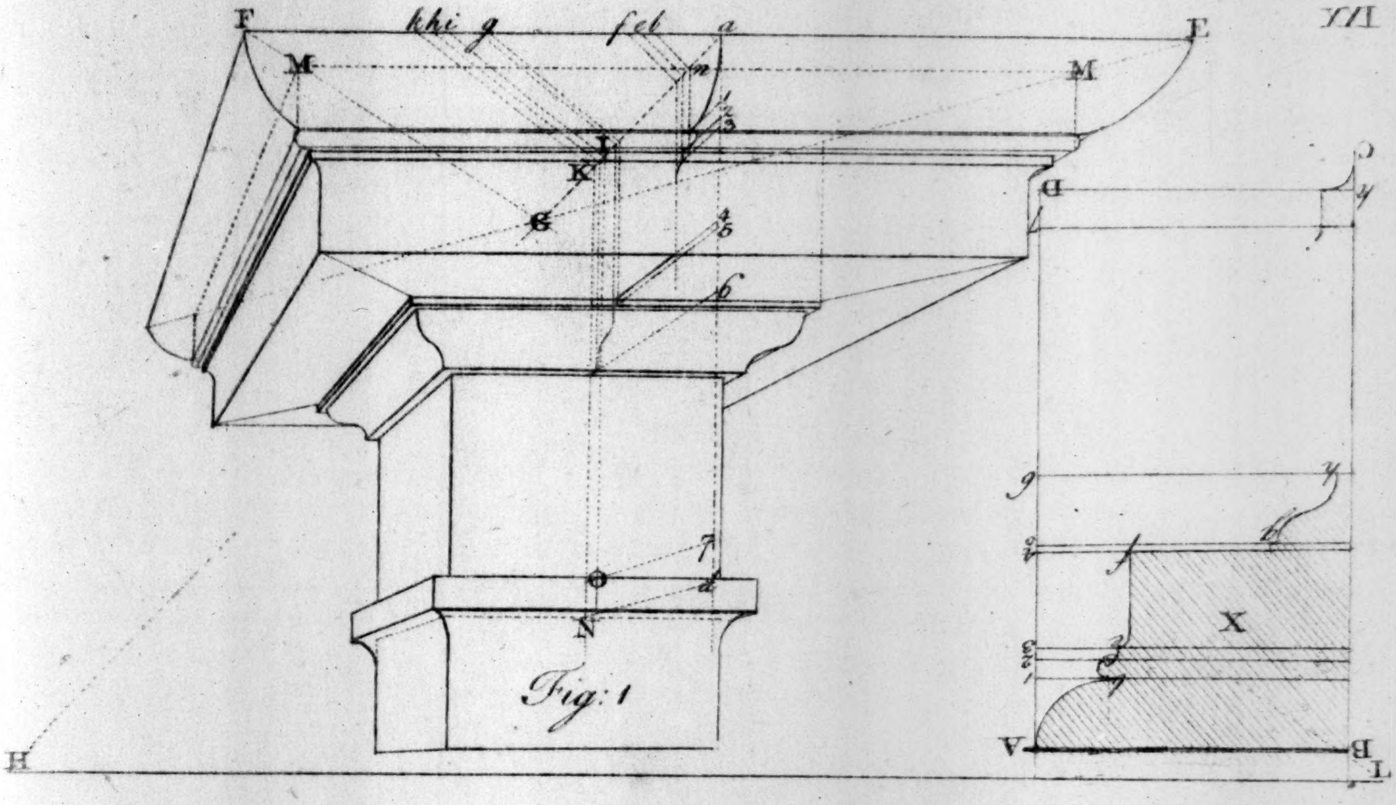
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Fig: 1

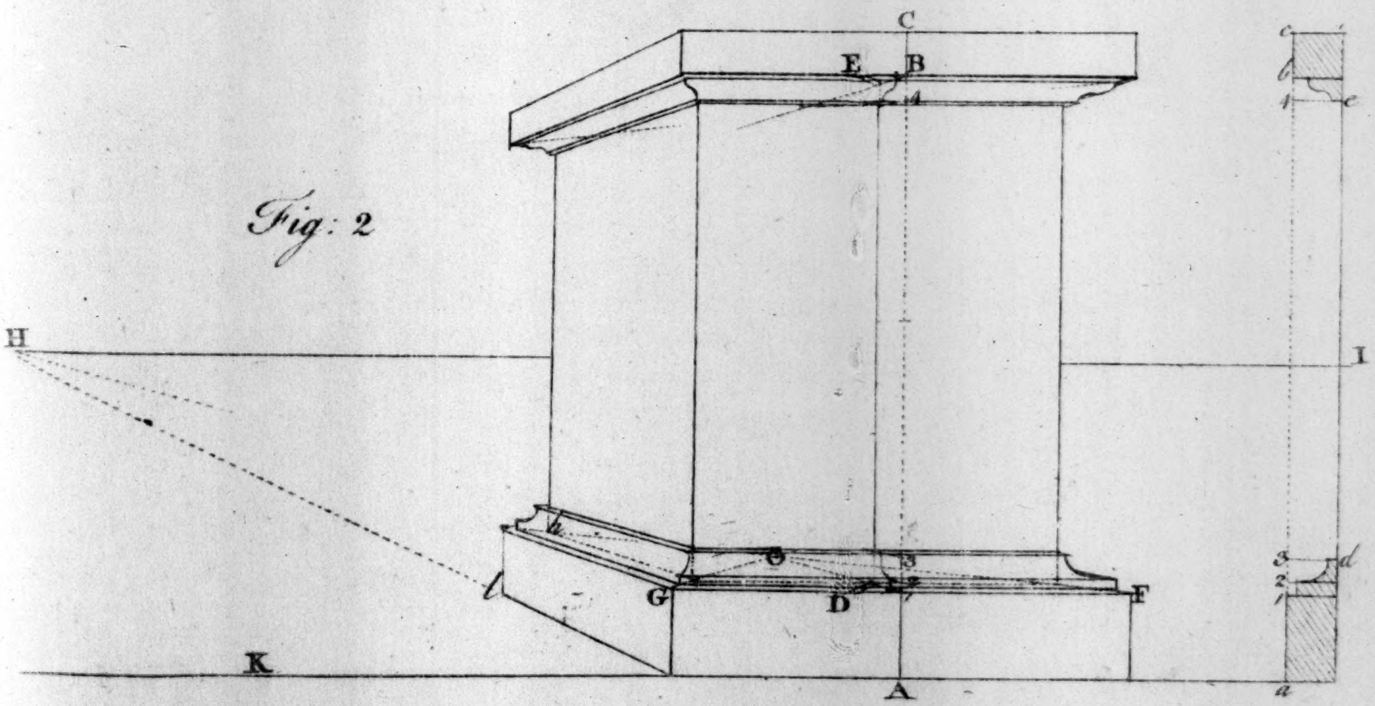
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Fig: 2

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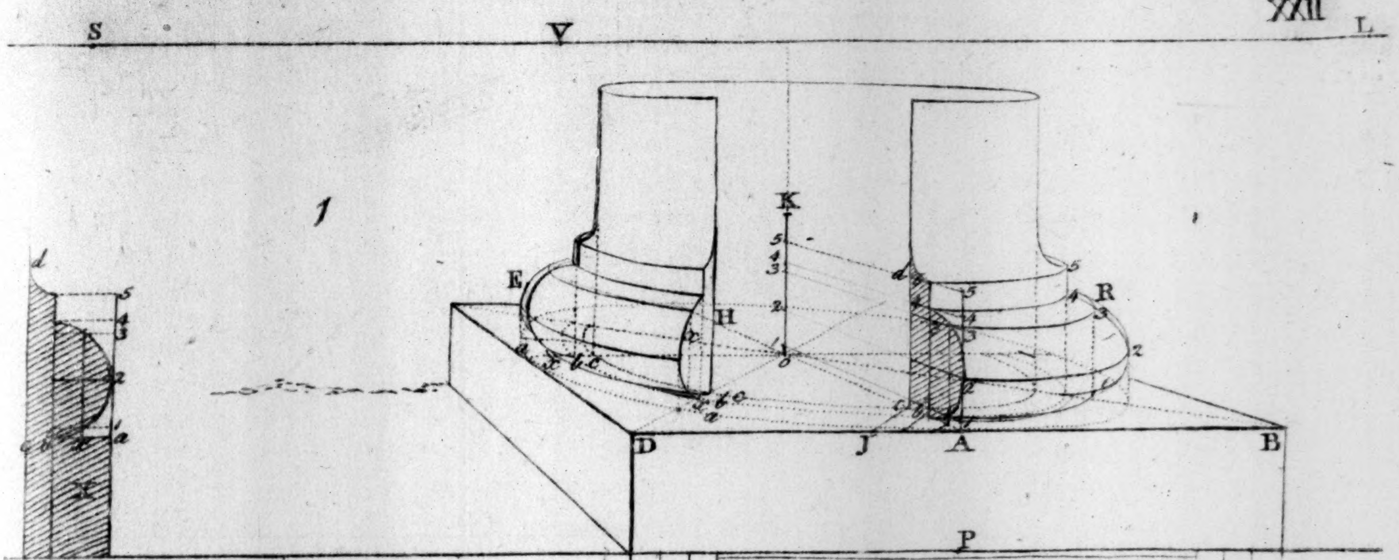
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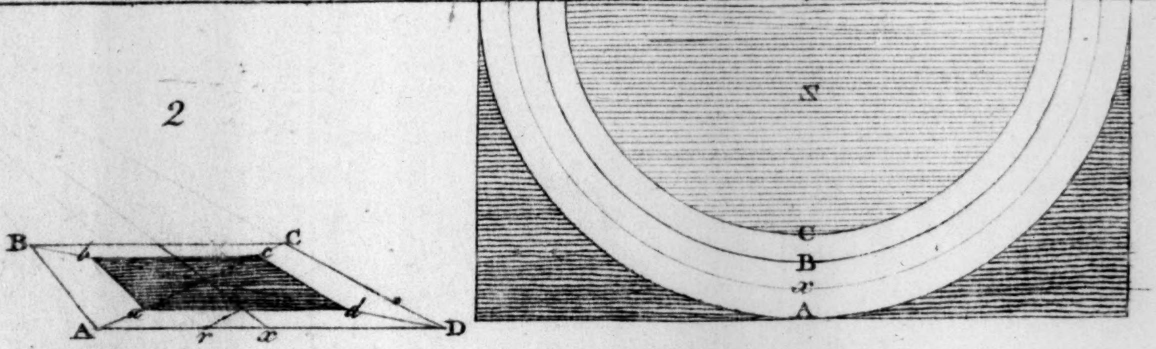
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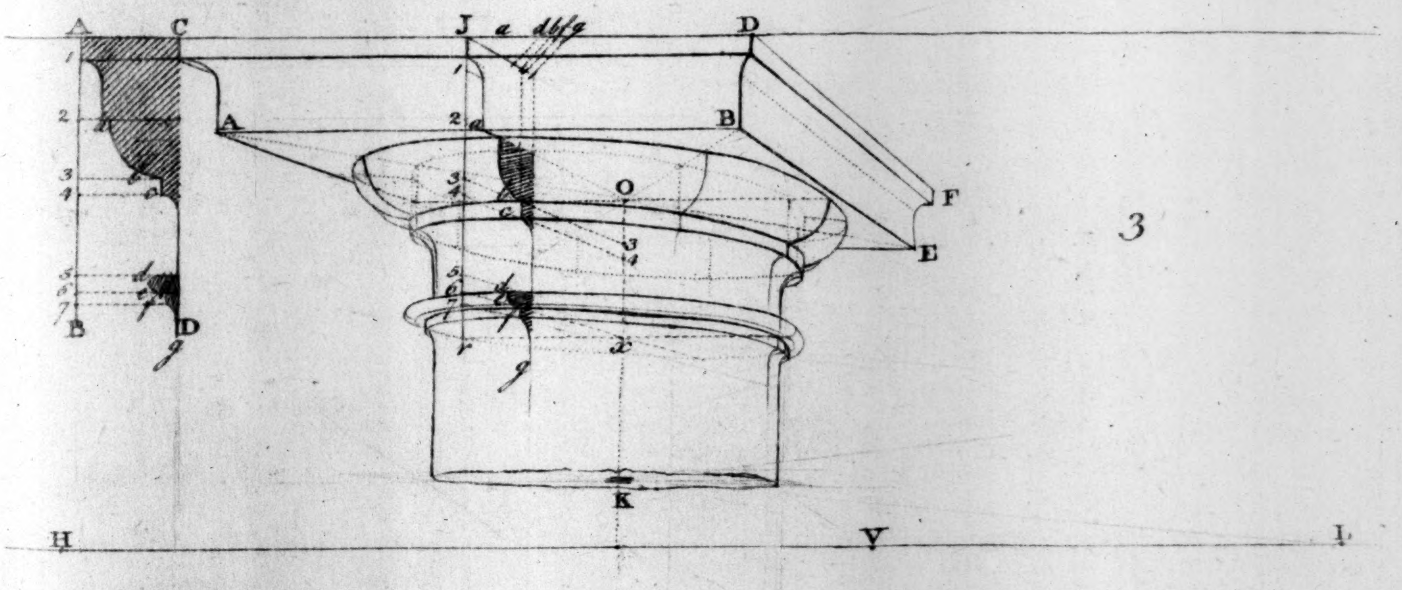
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BRITISH
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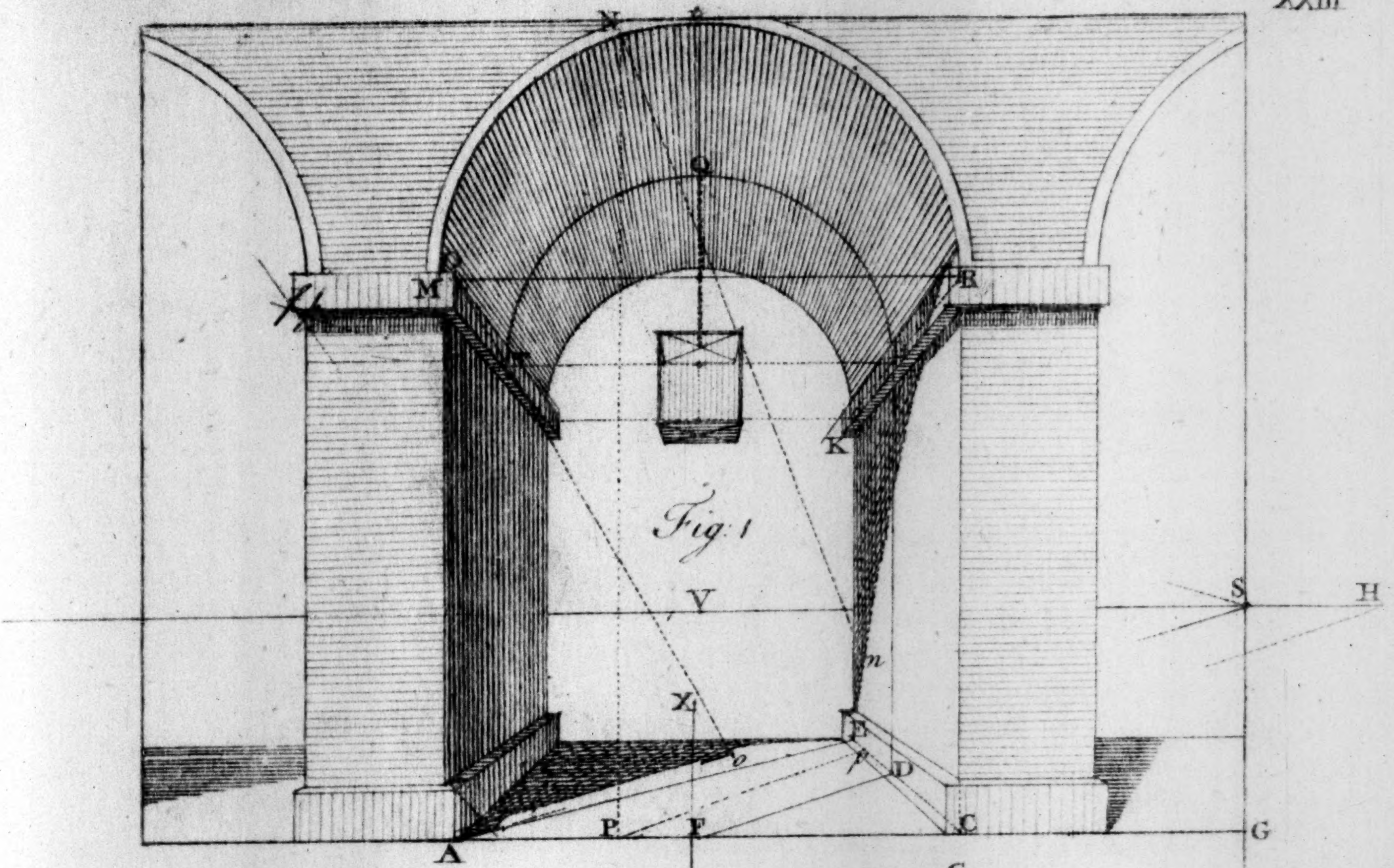
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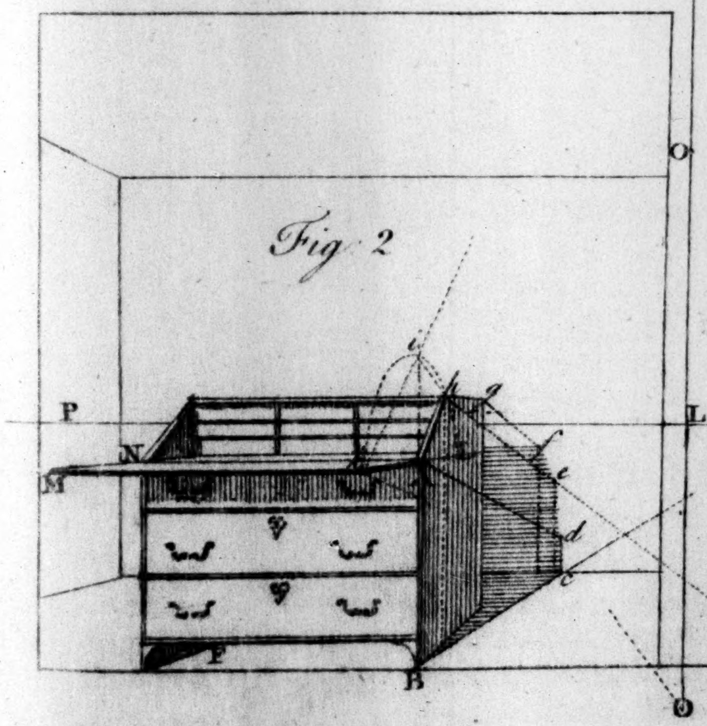
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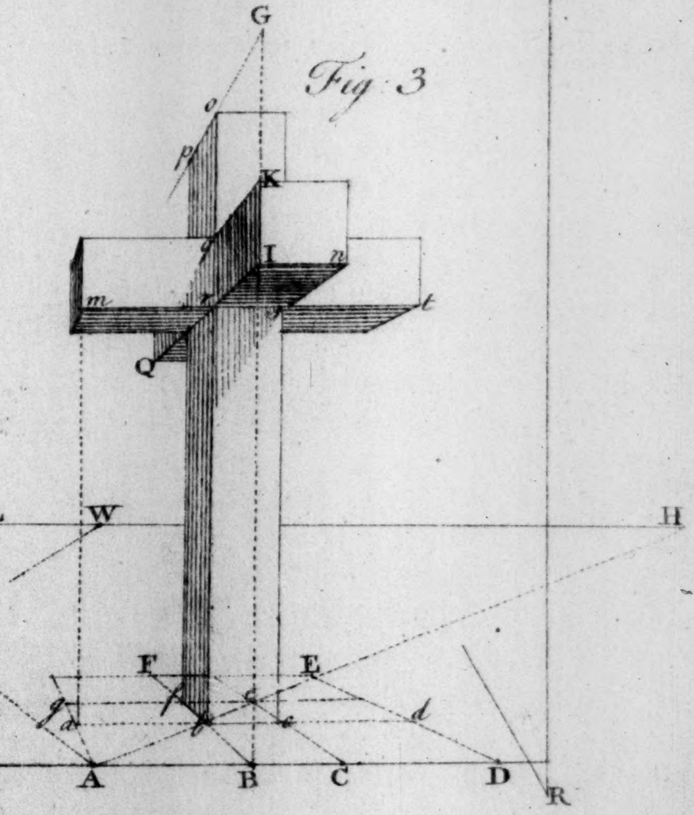
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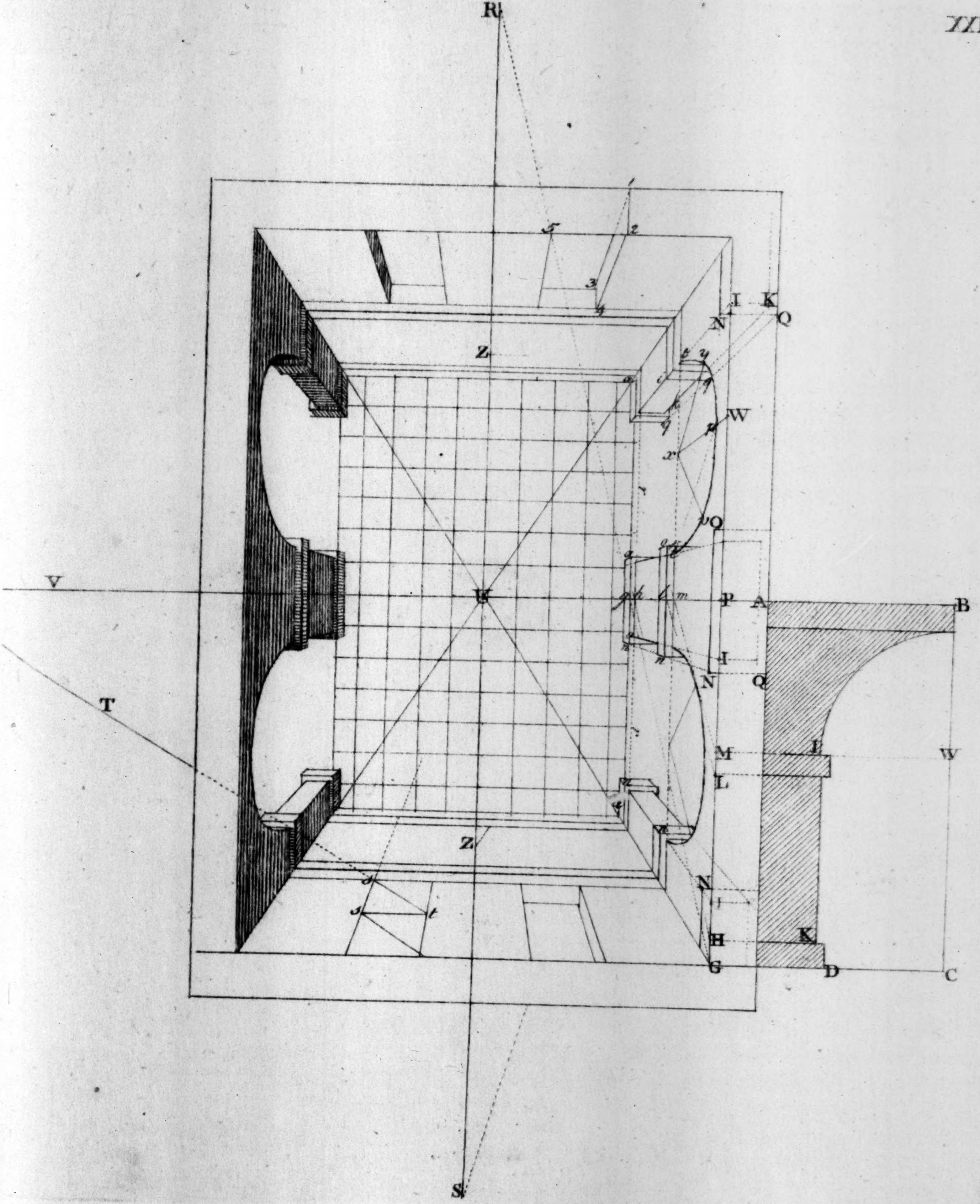
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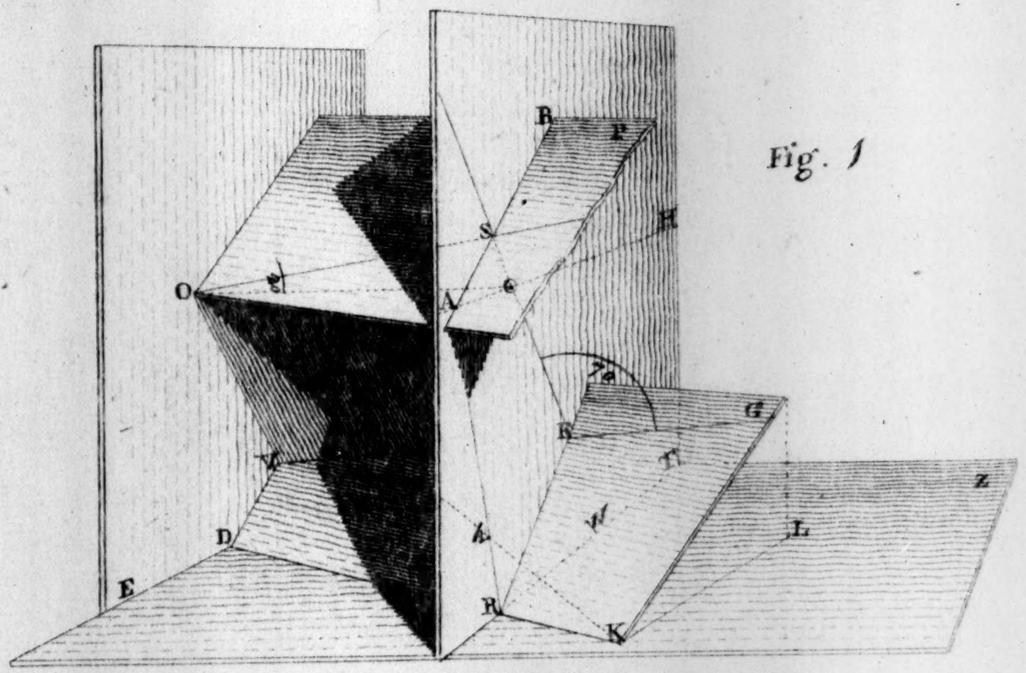

Fig. 1

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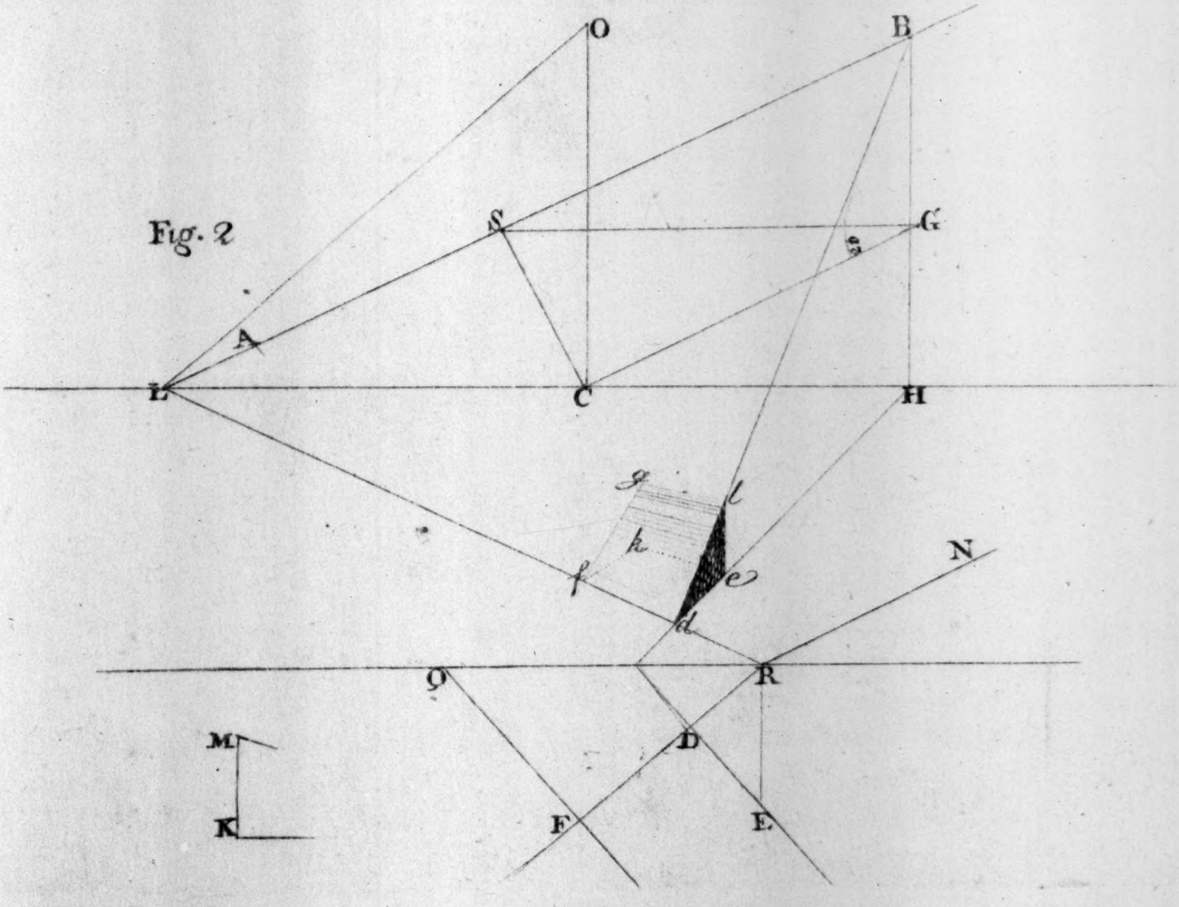

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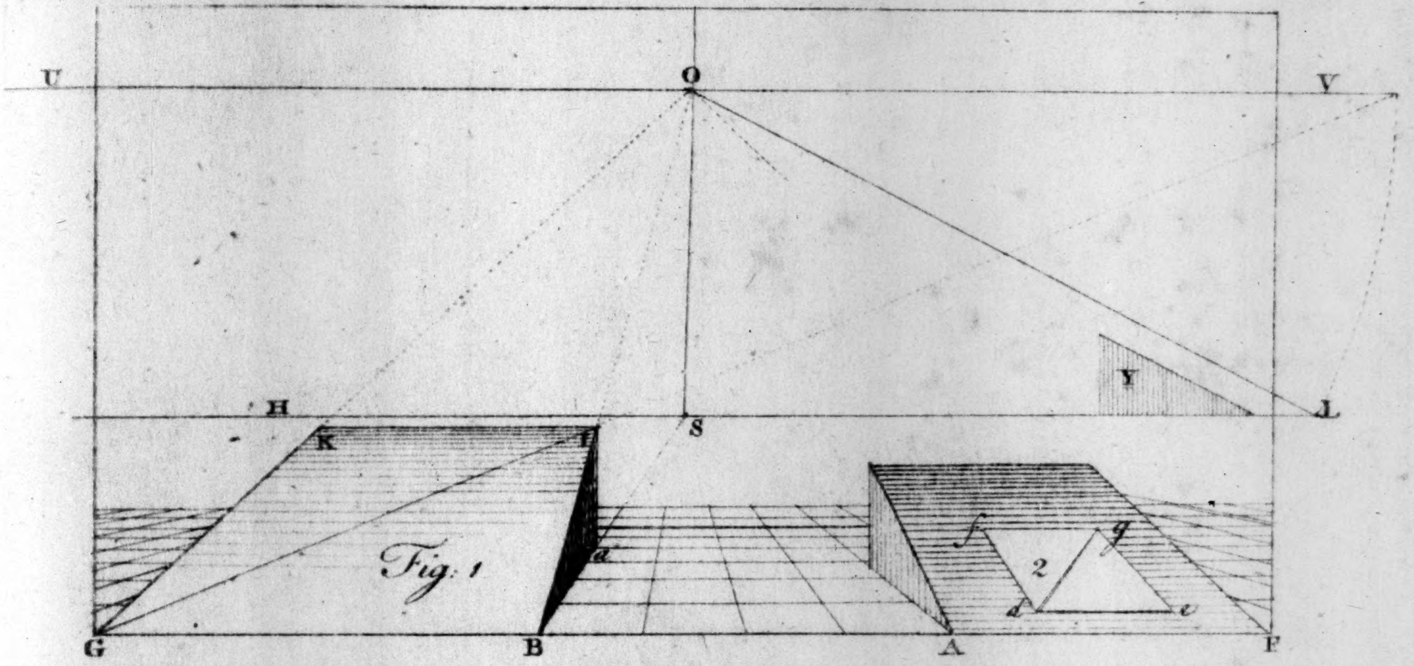
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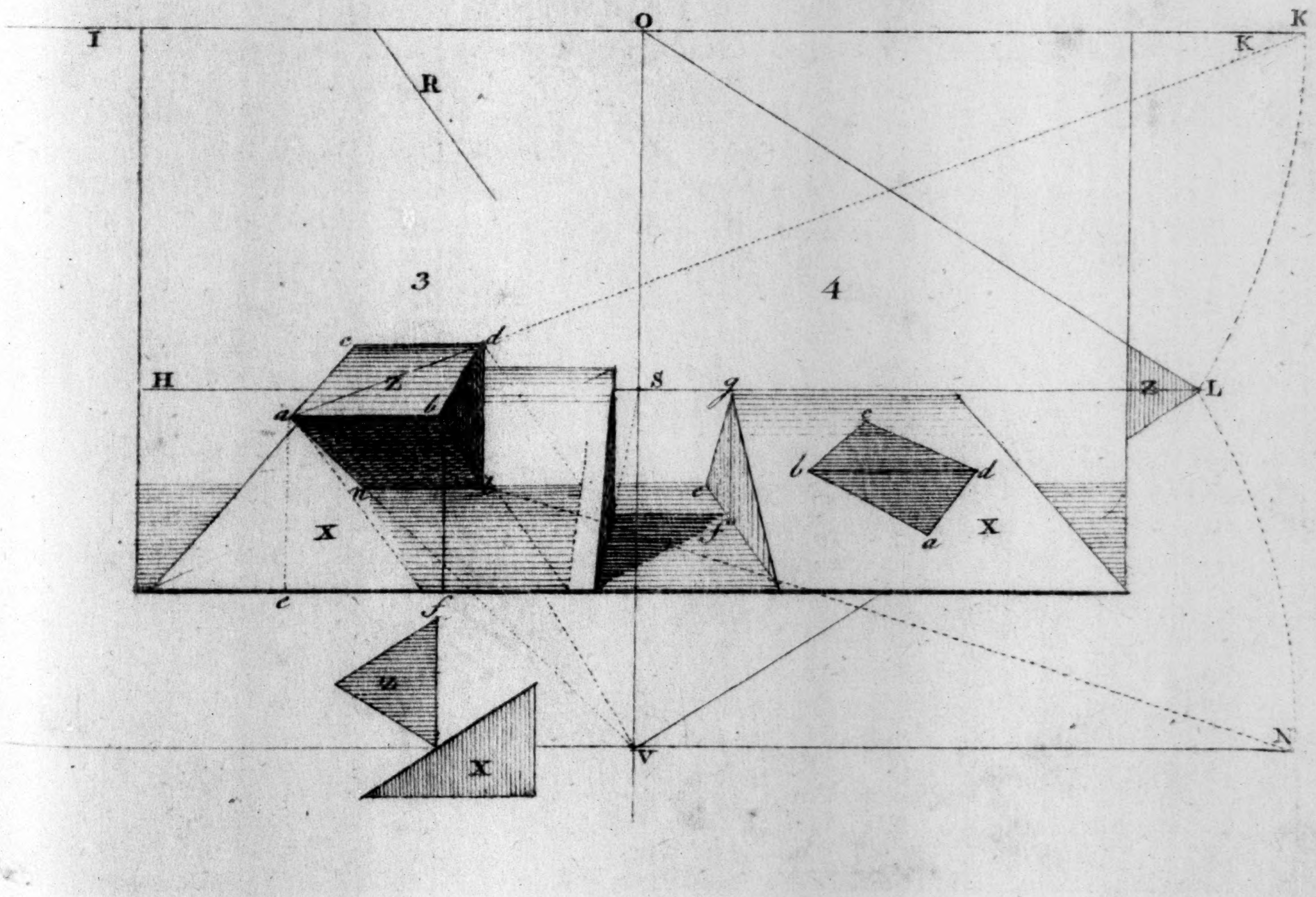
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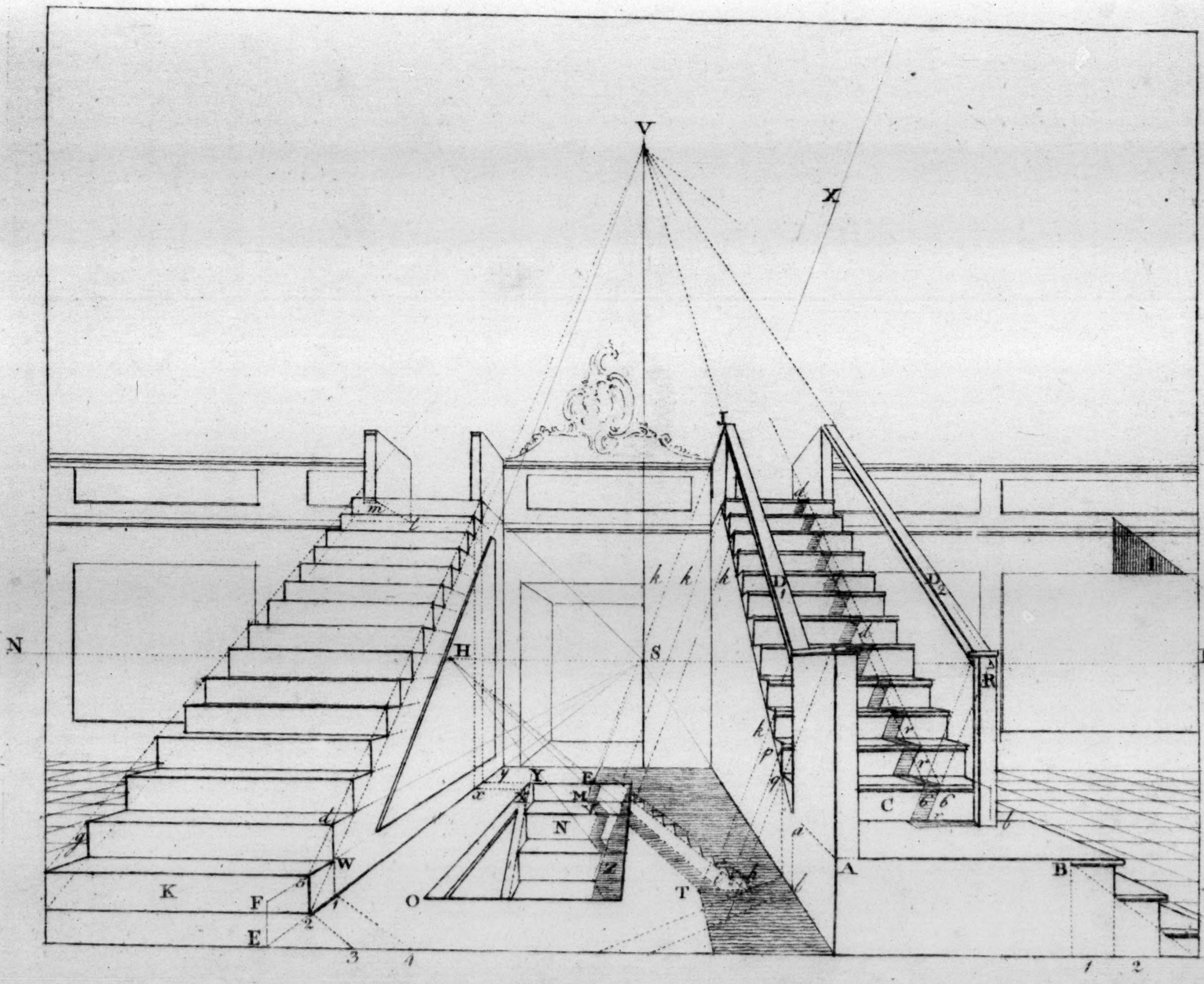
BRITISH 3 MY 76 MUSEUM



XXVII

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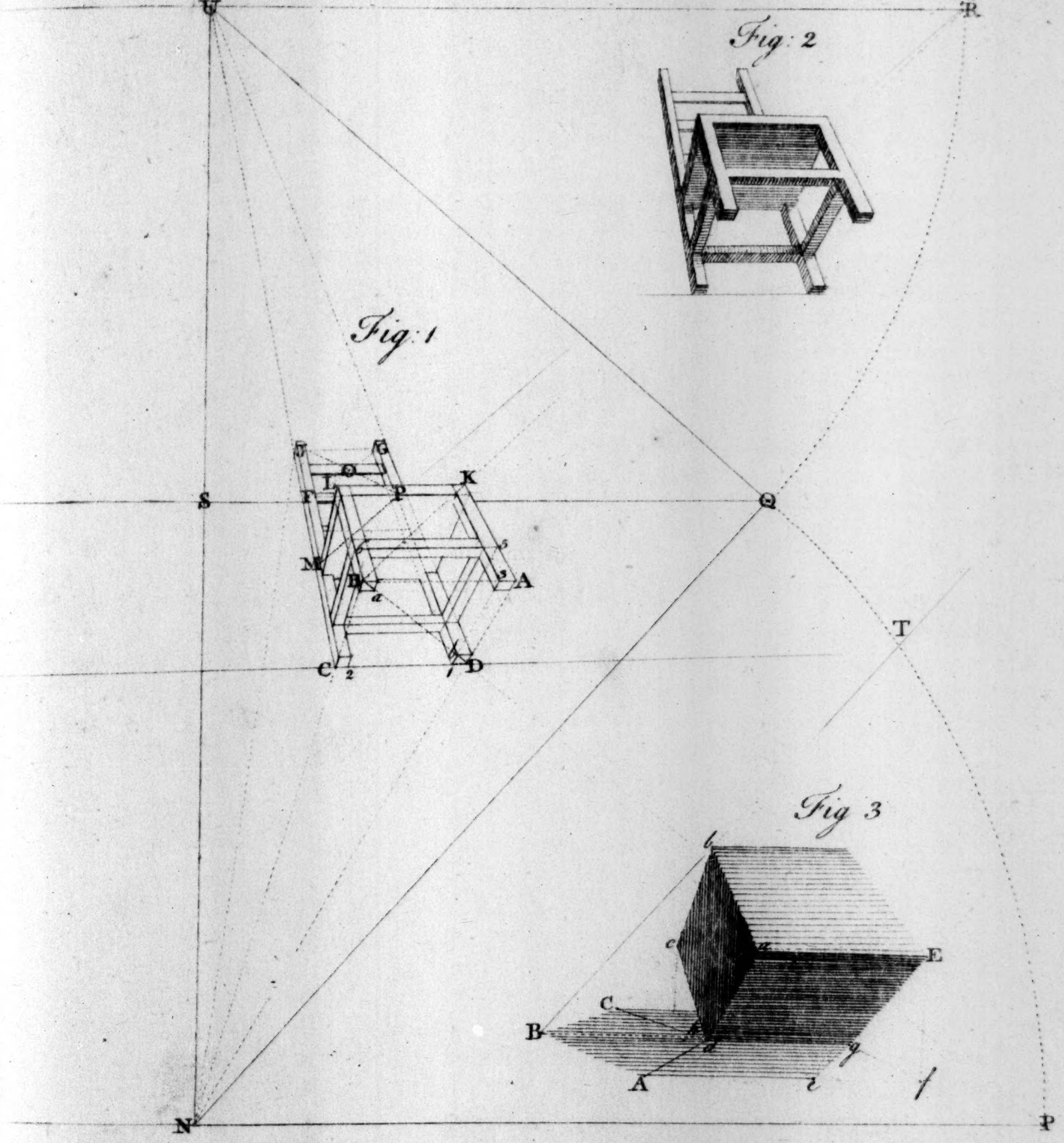

Fig: 1

Fig: 2

Fig 3

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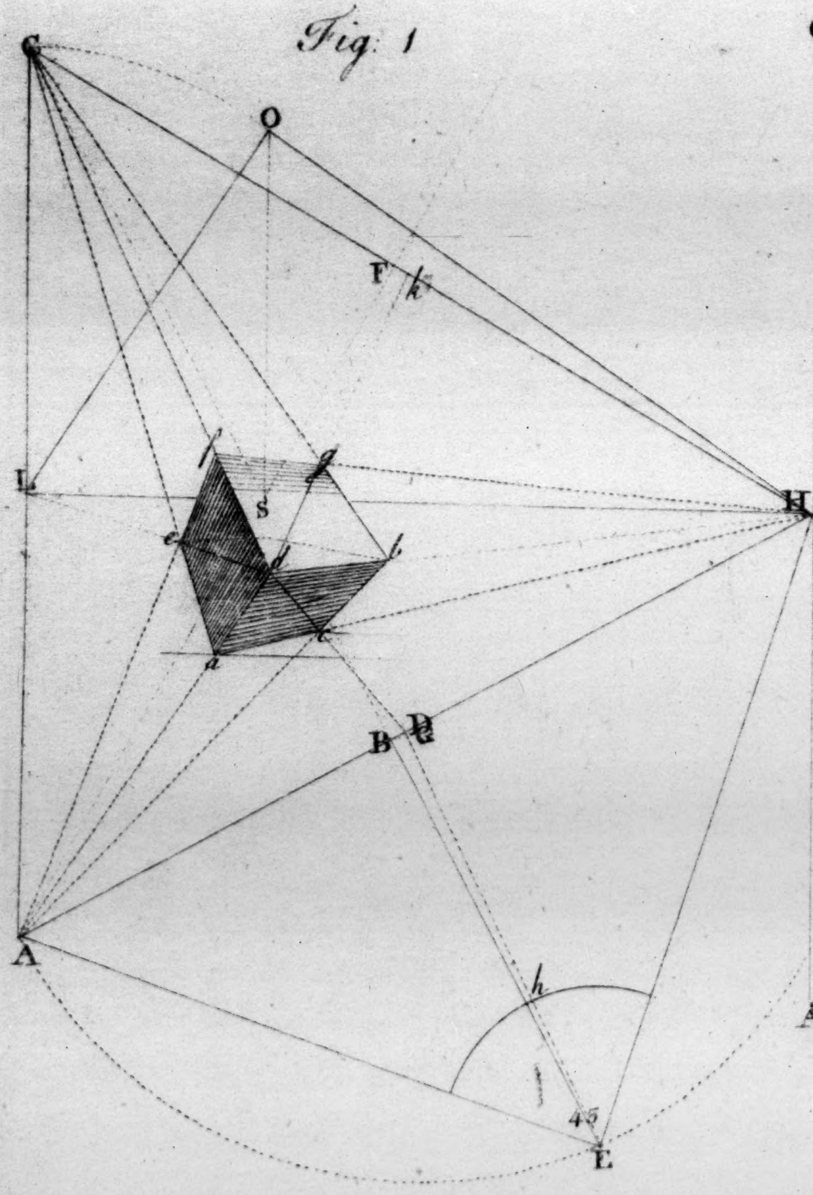
Fig: 1



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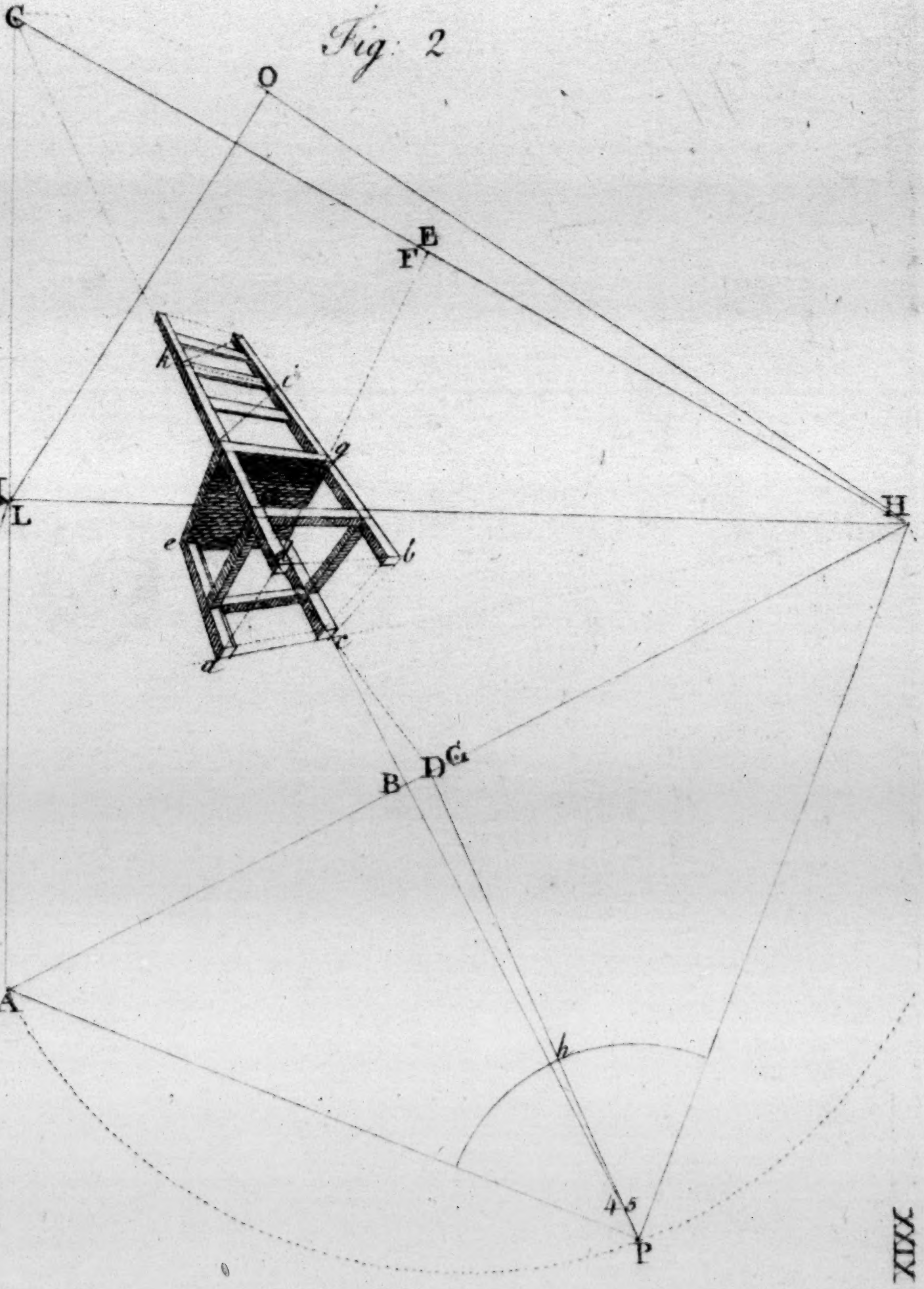

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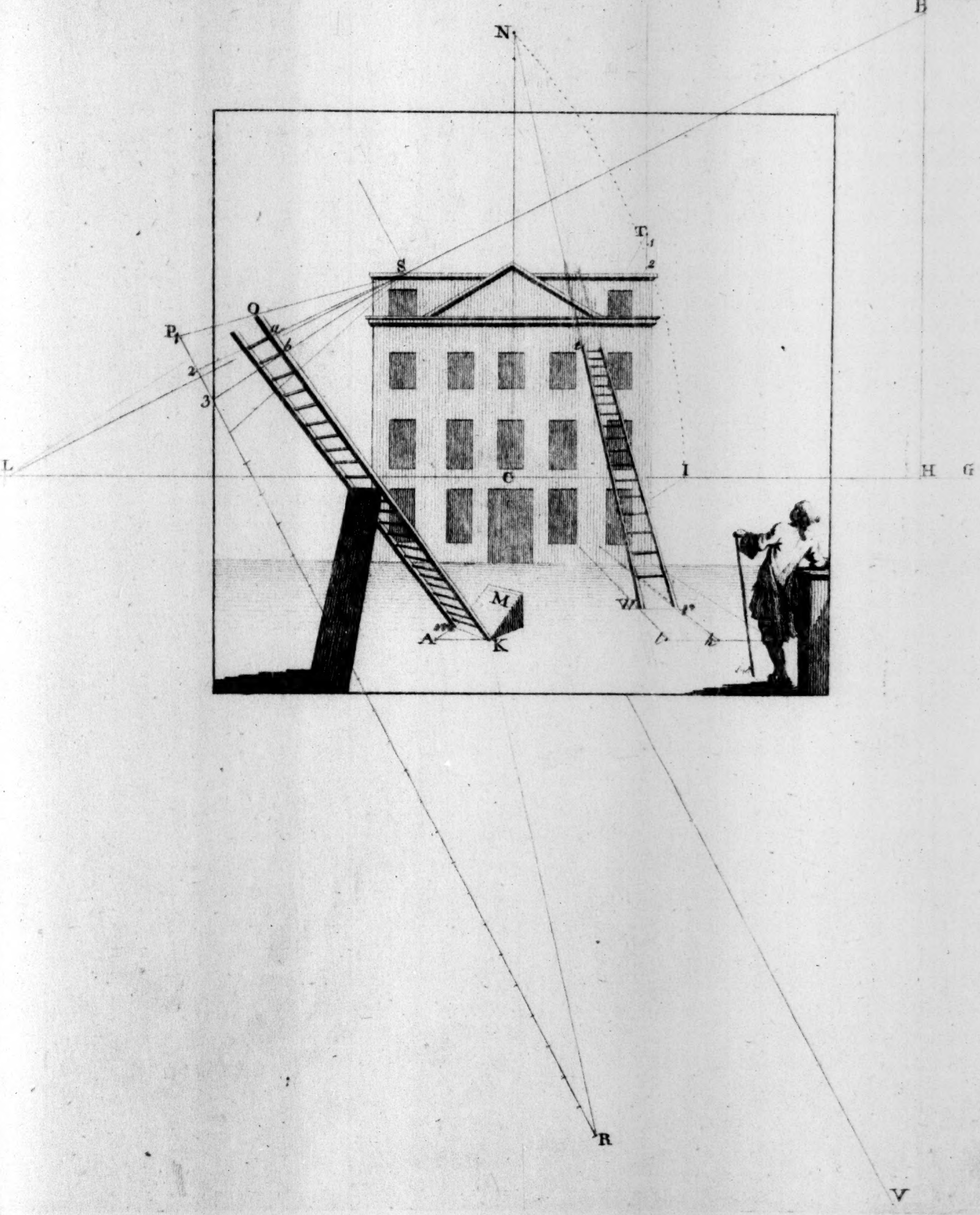

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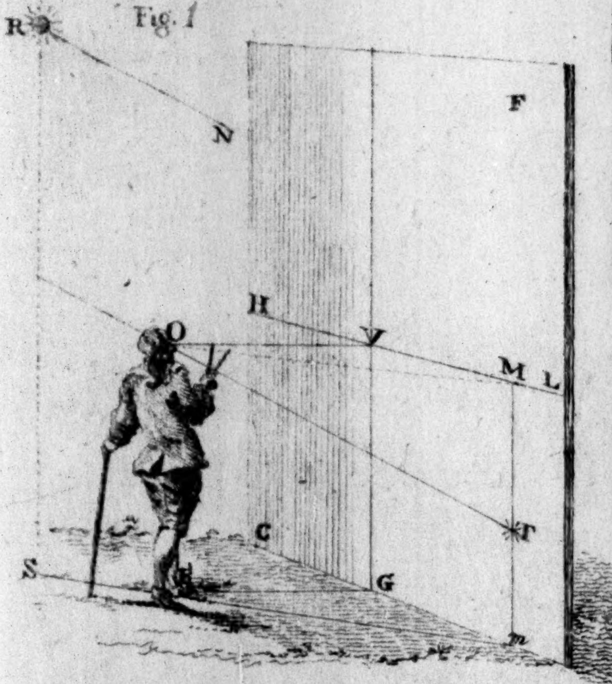

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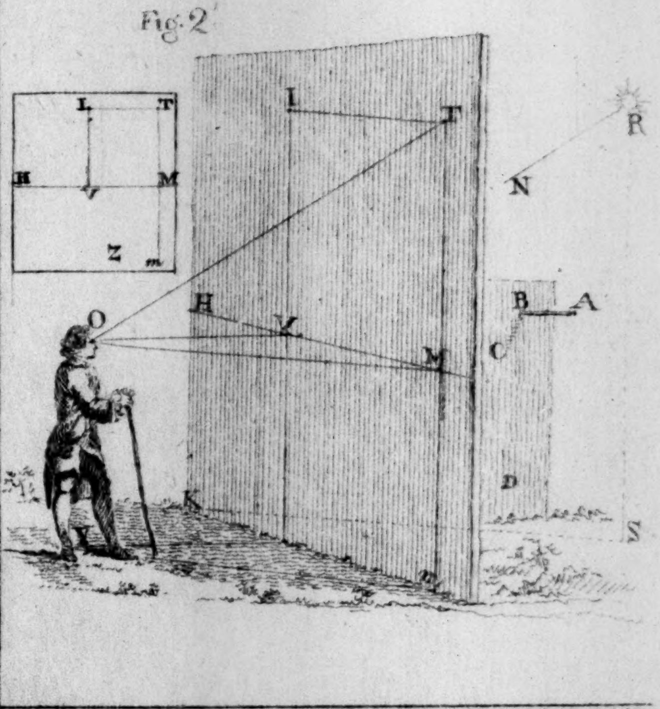

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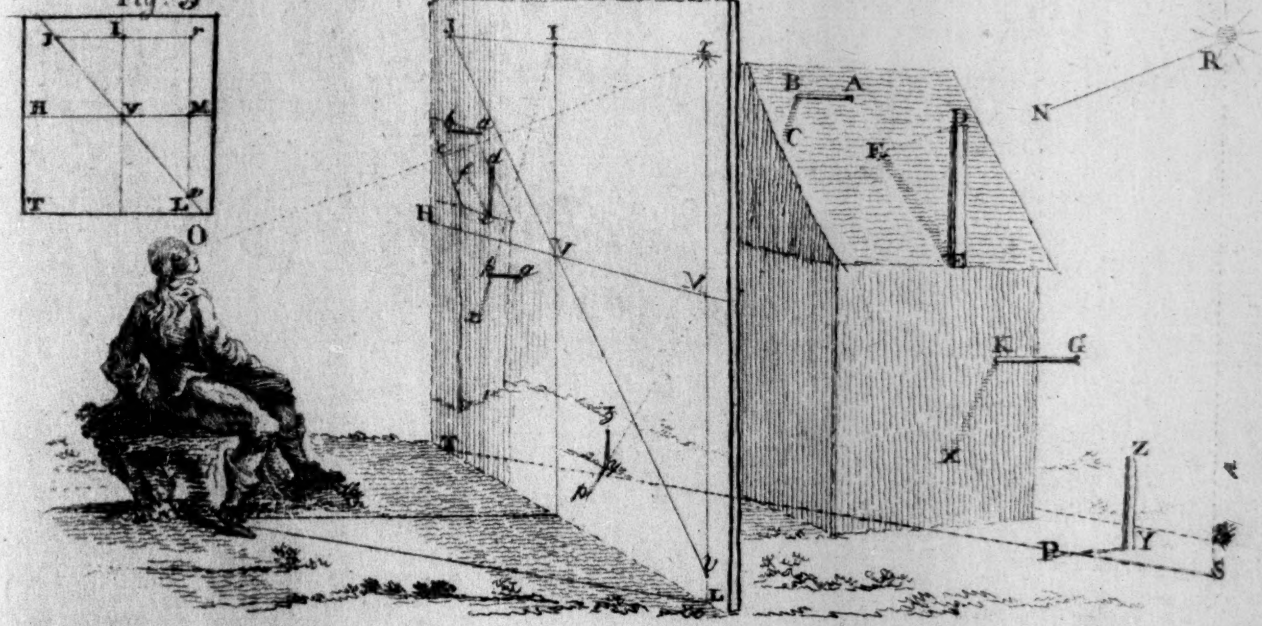

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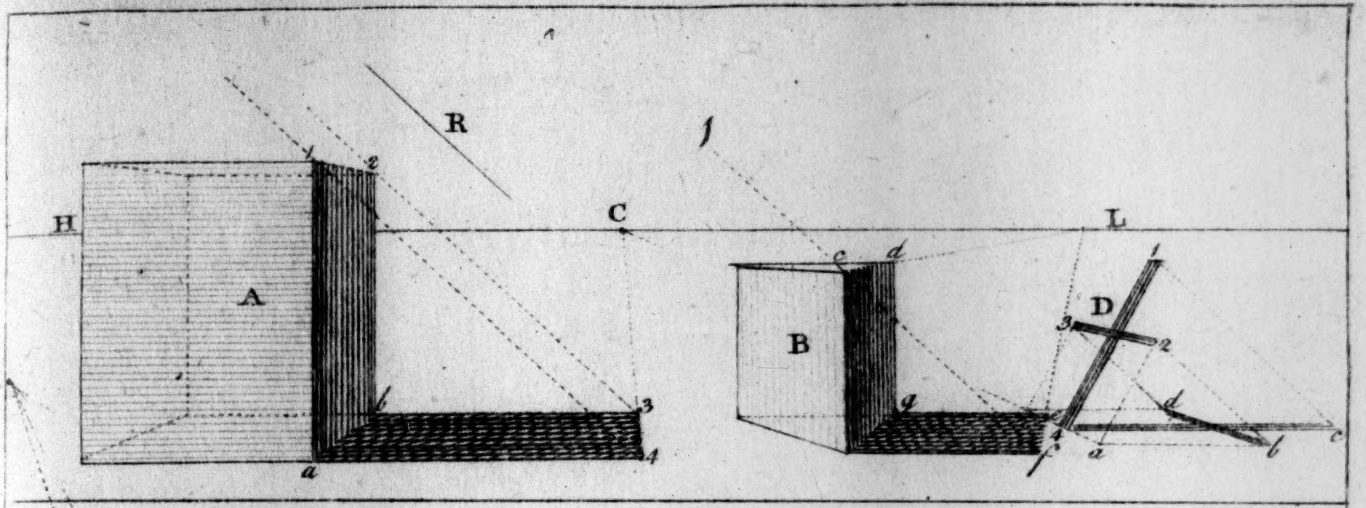
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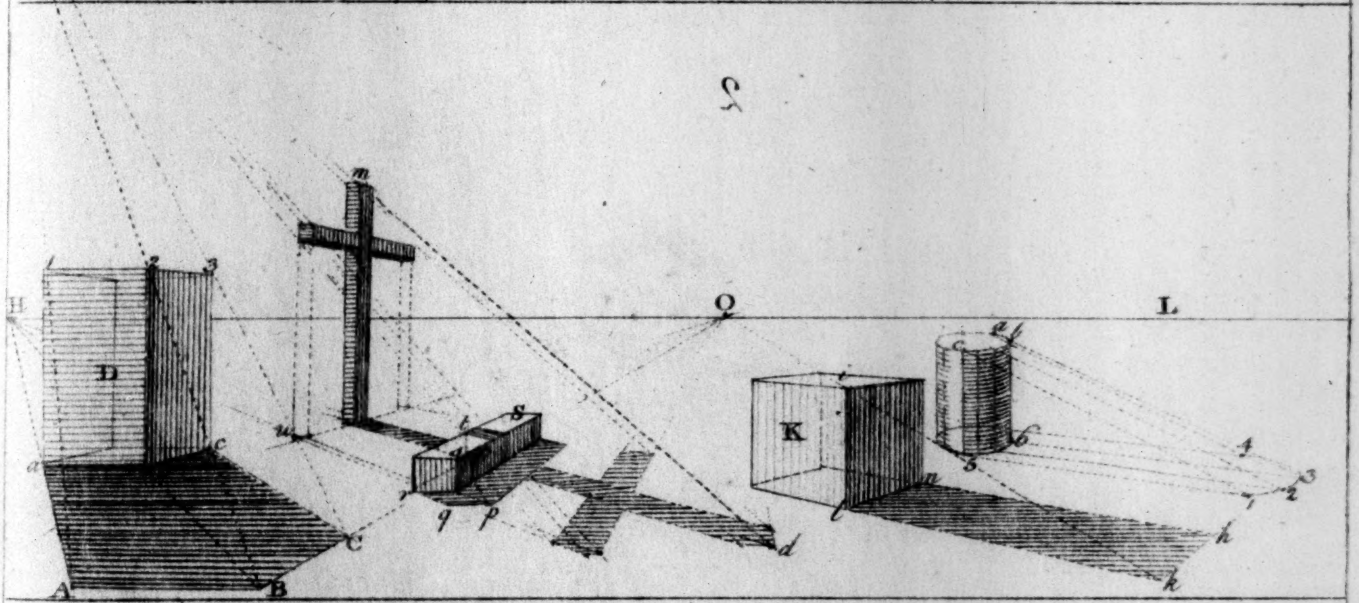
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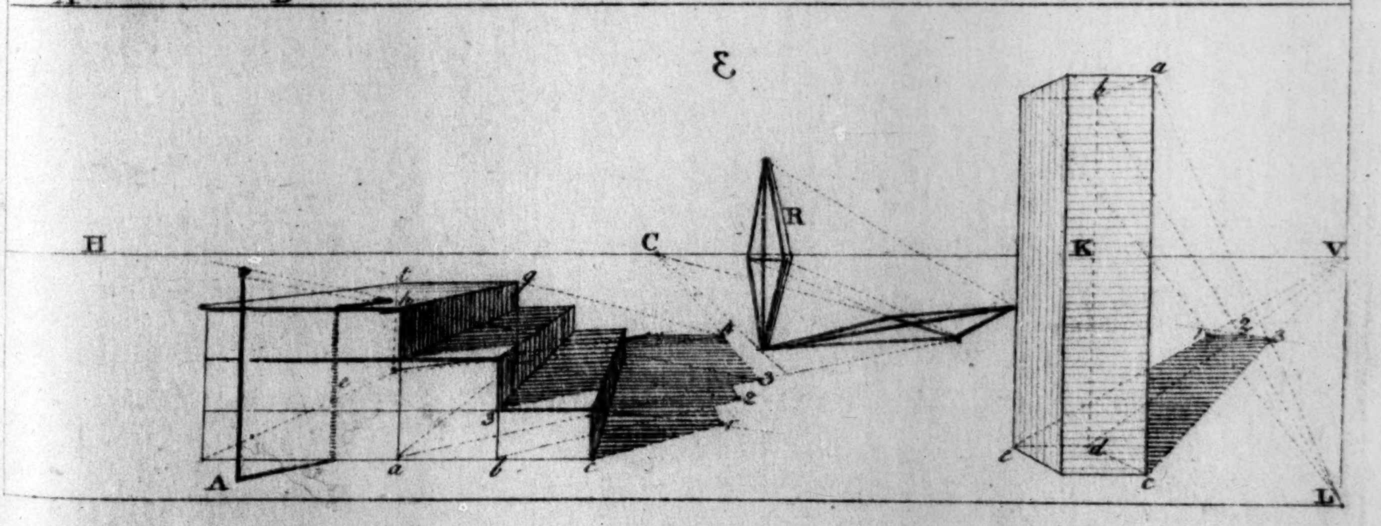
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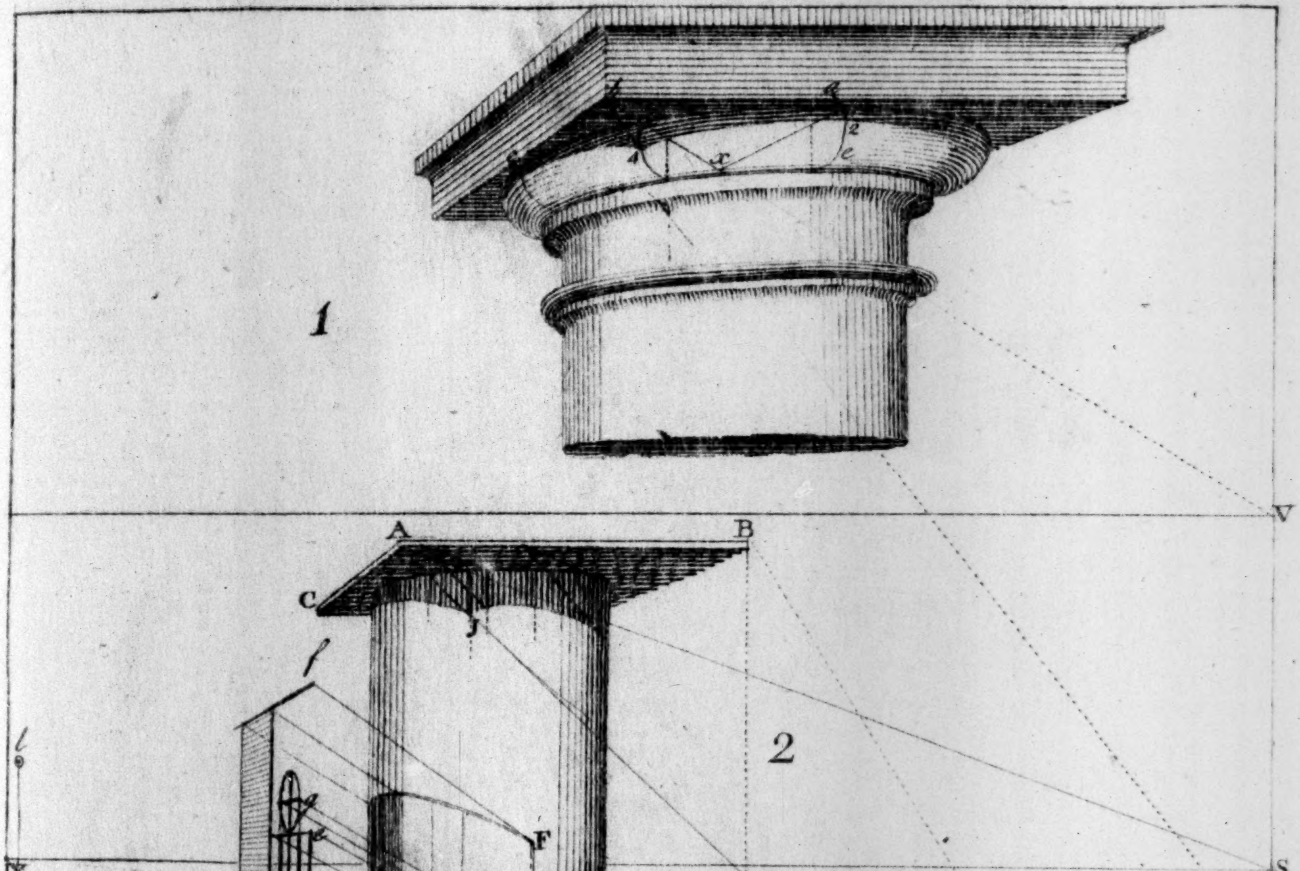
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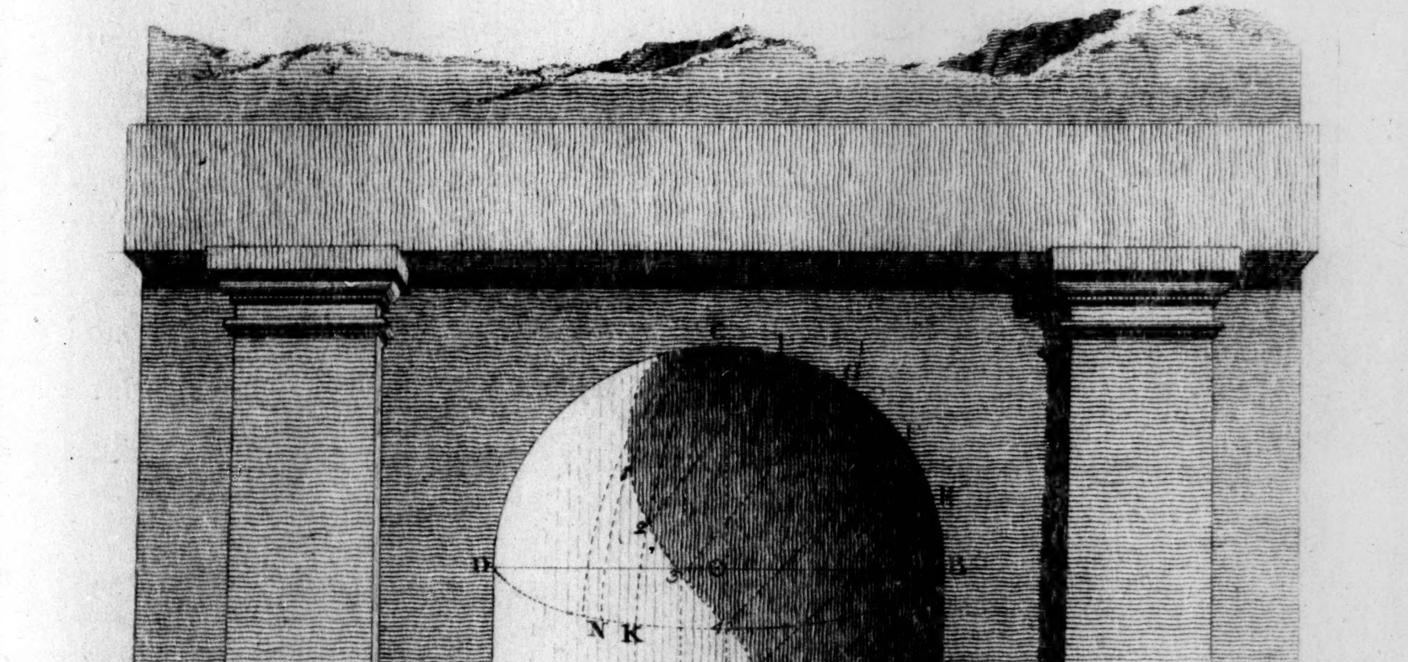
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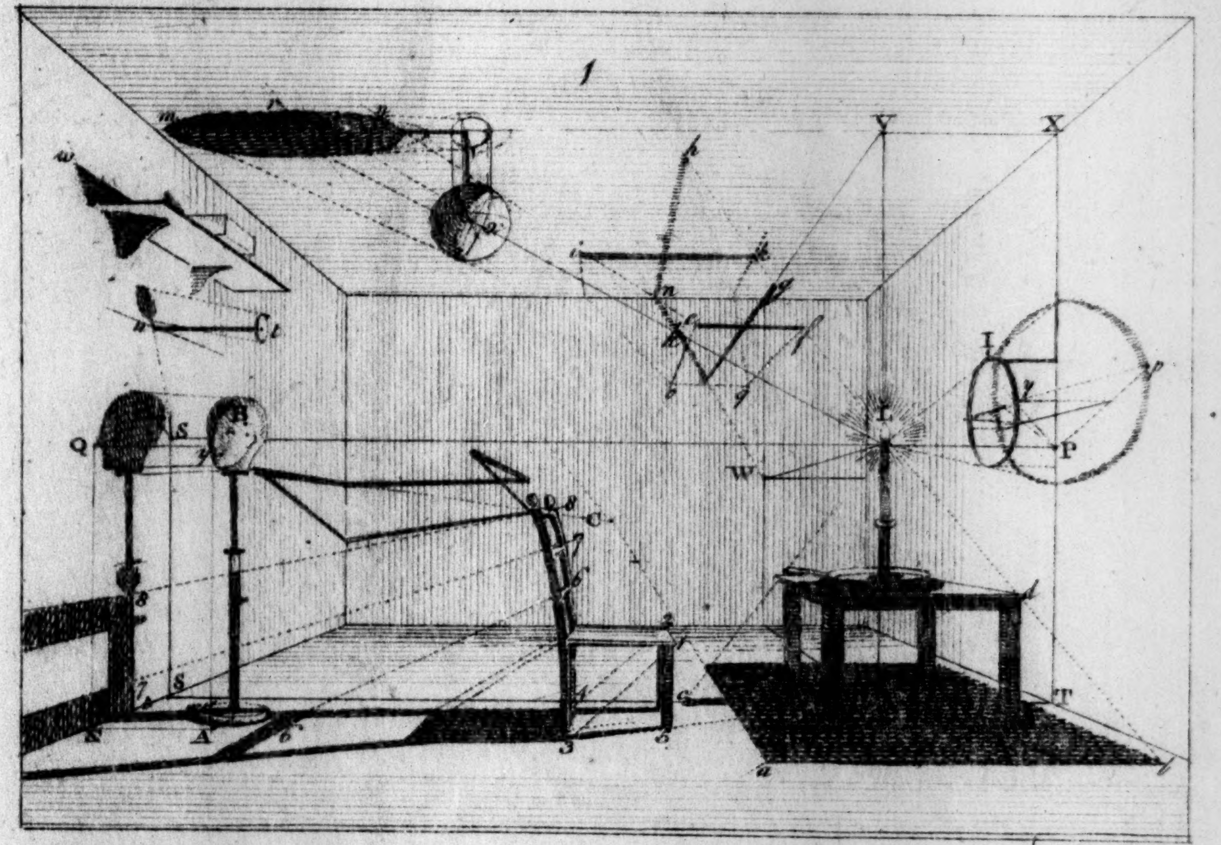

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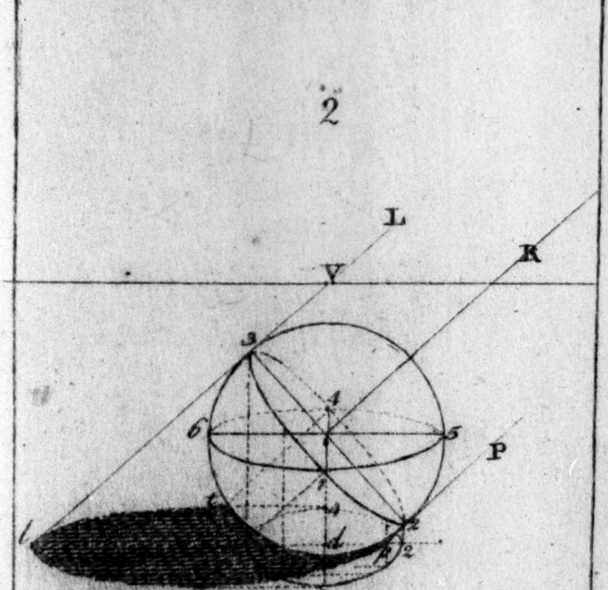

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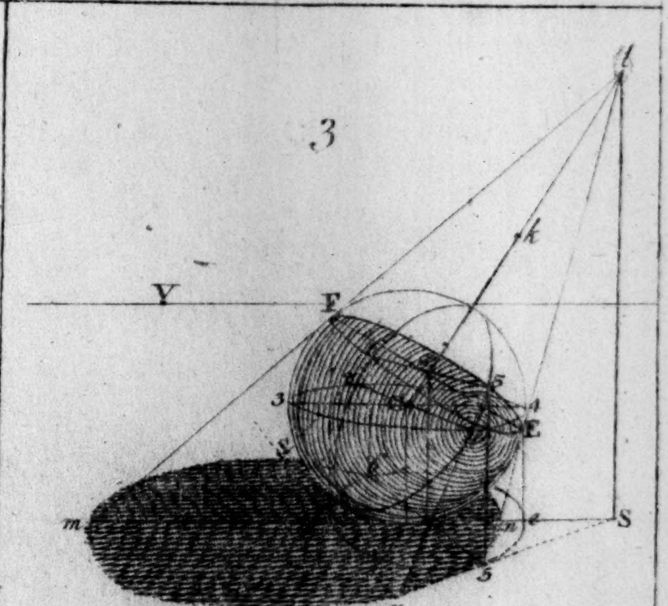

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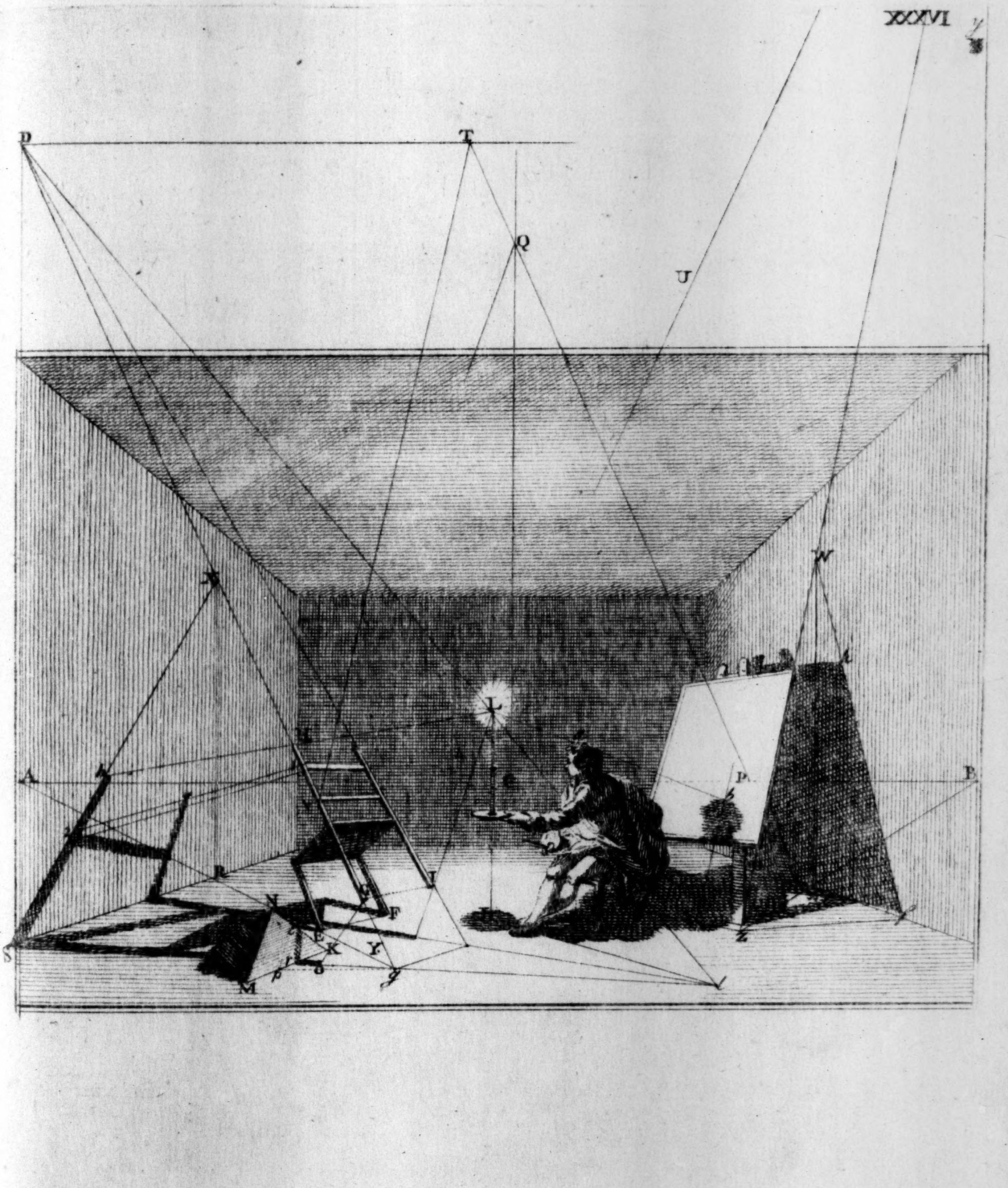

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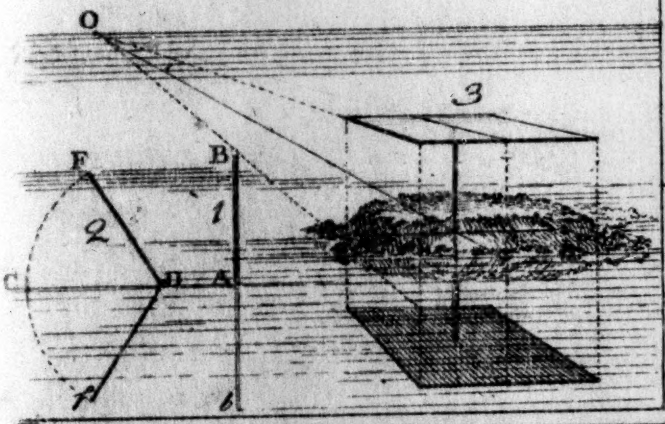
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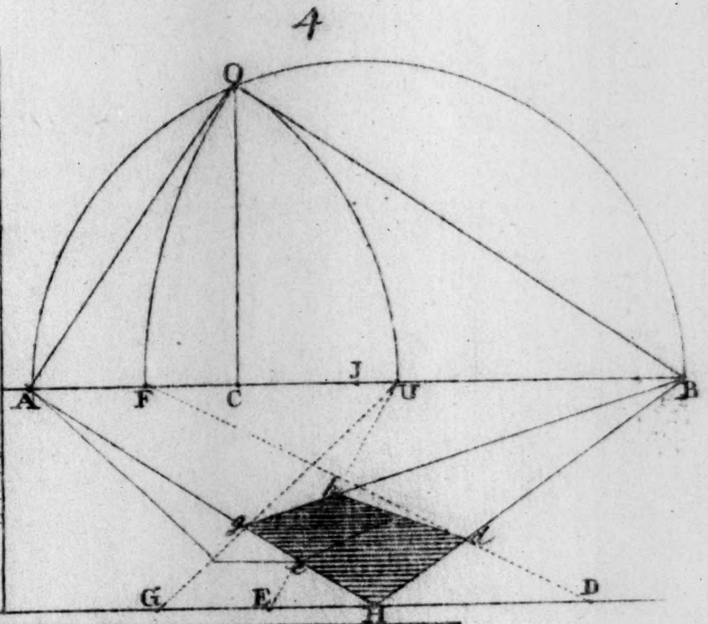
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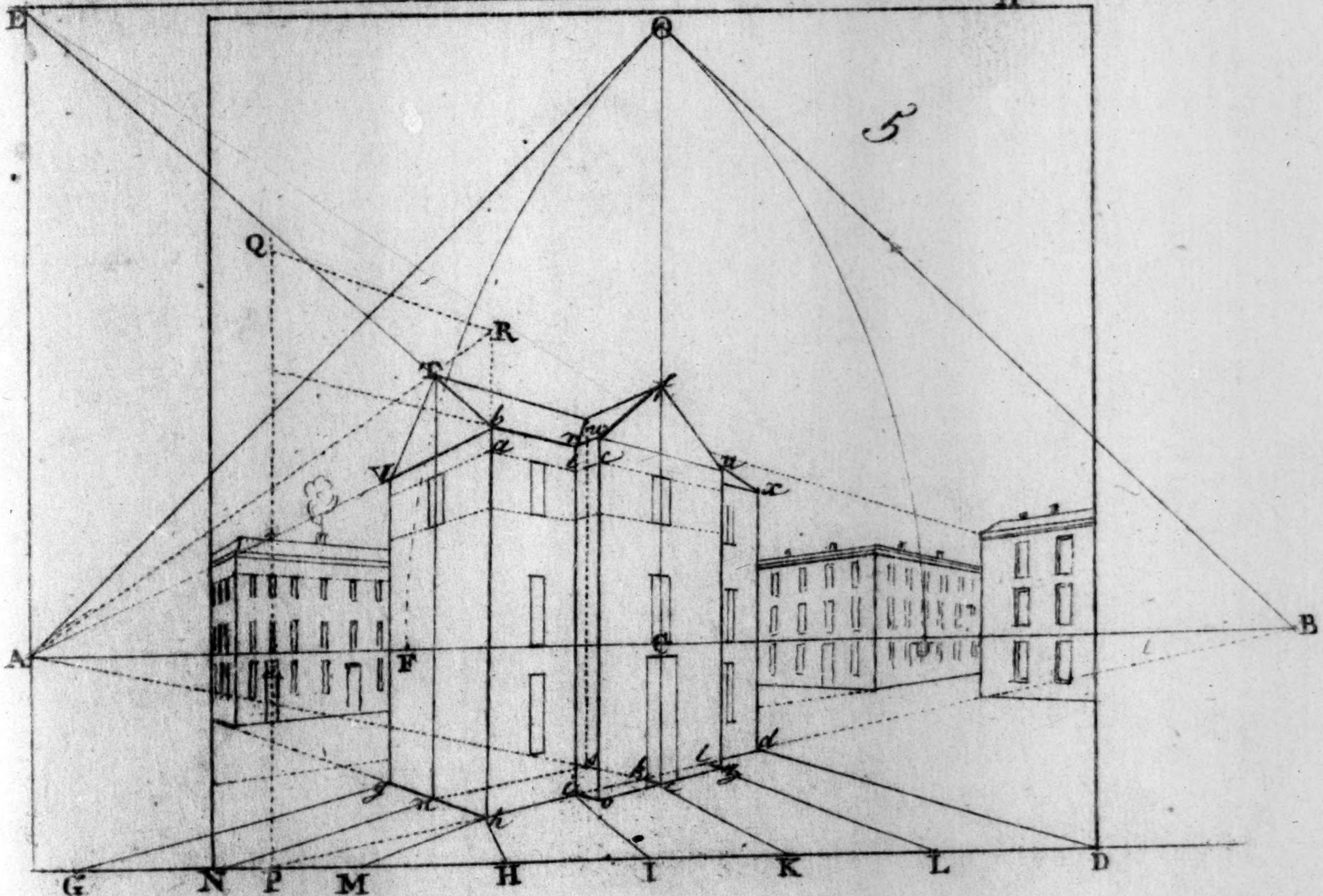
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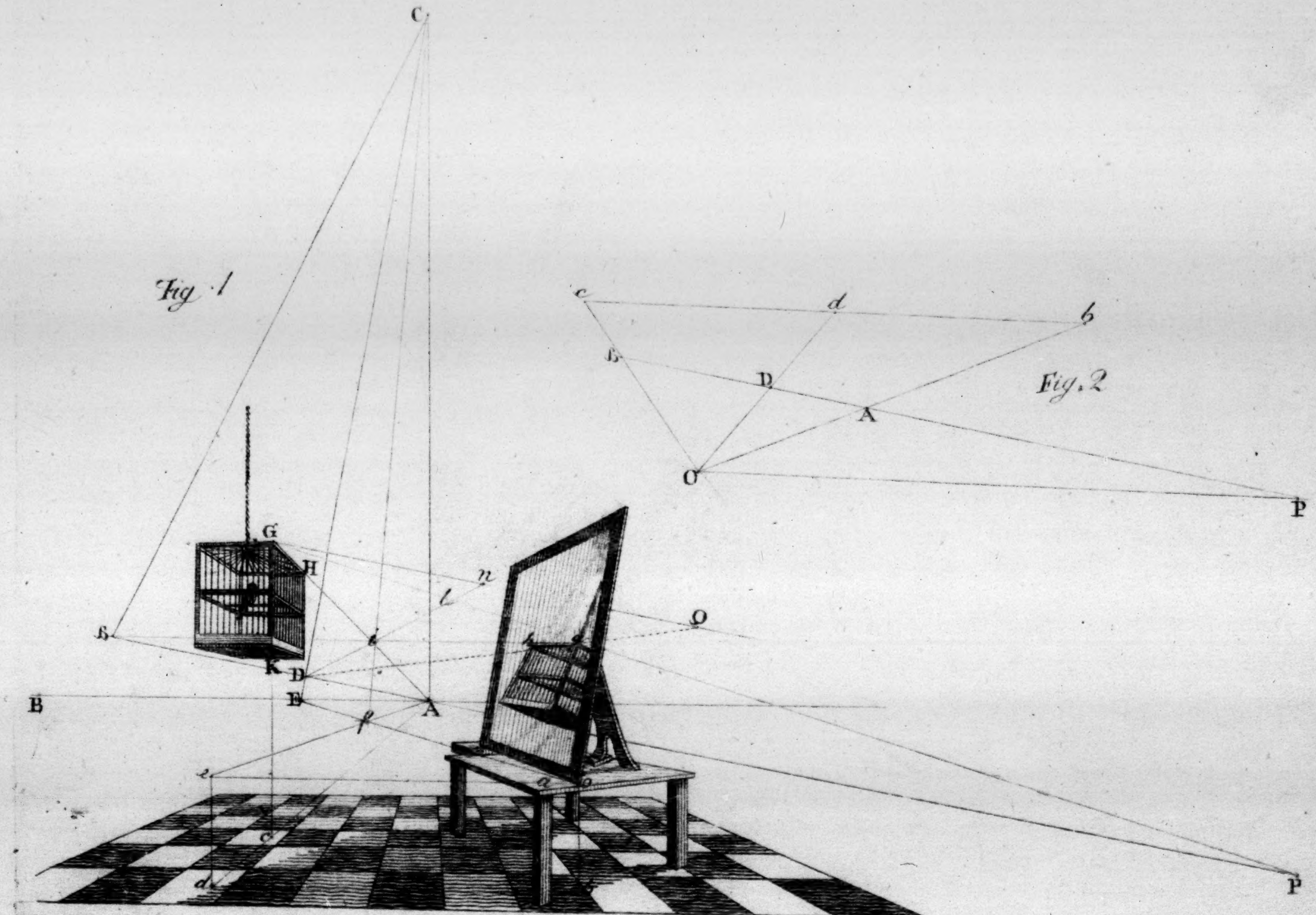
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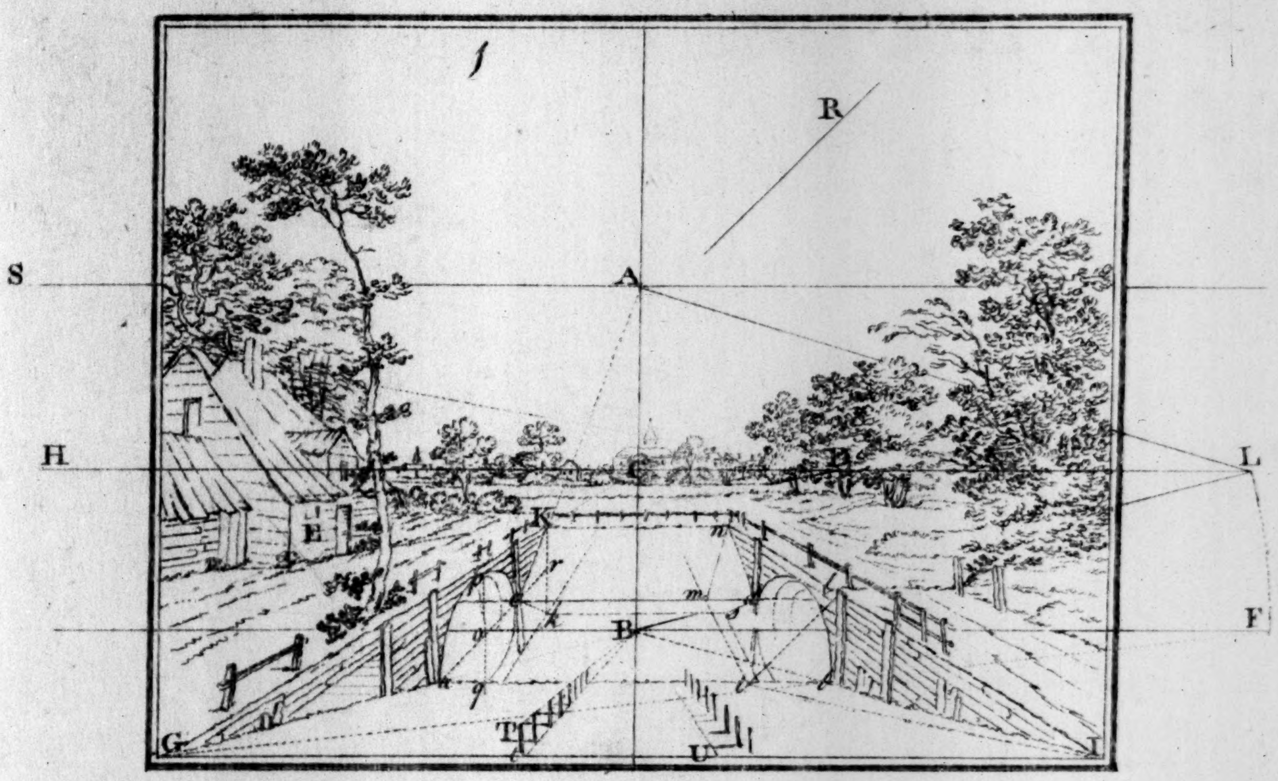
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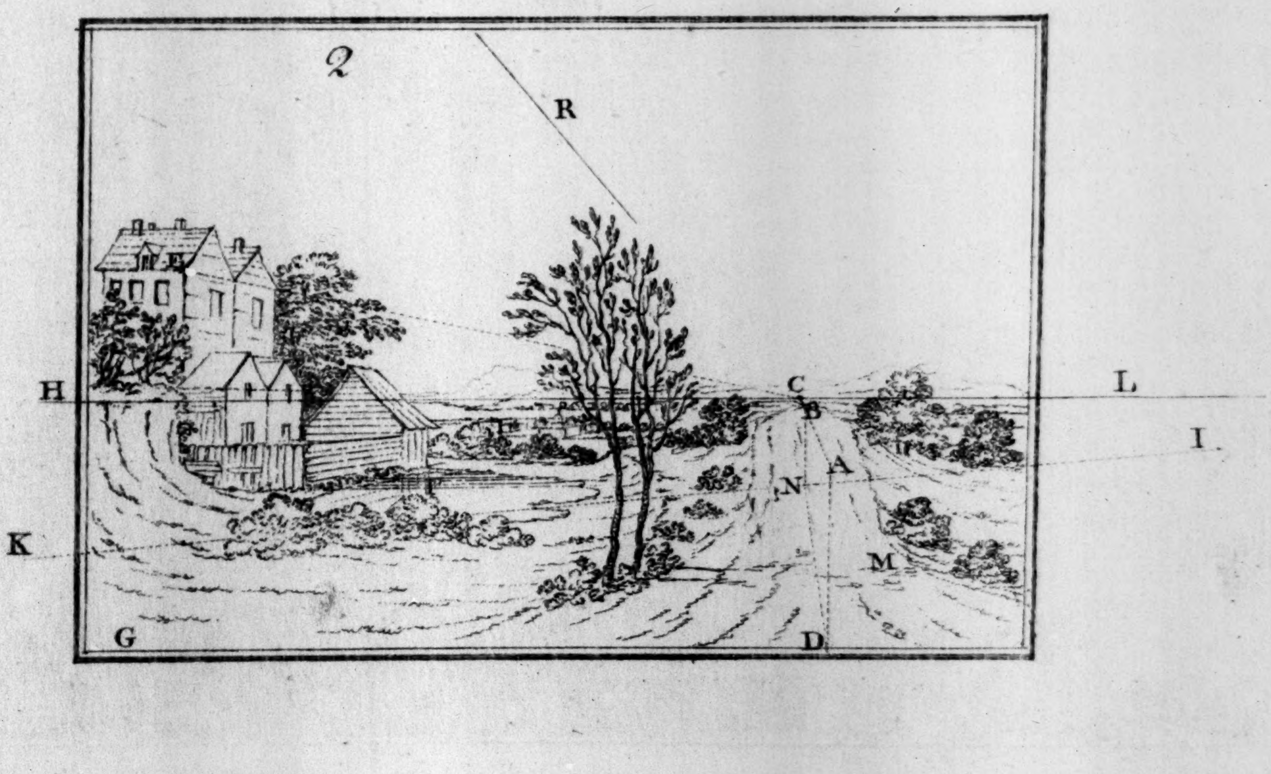
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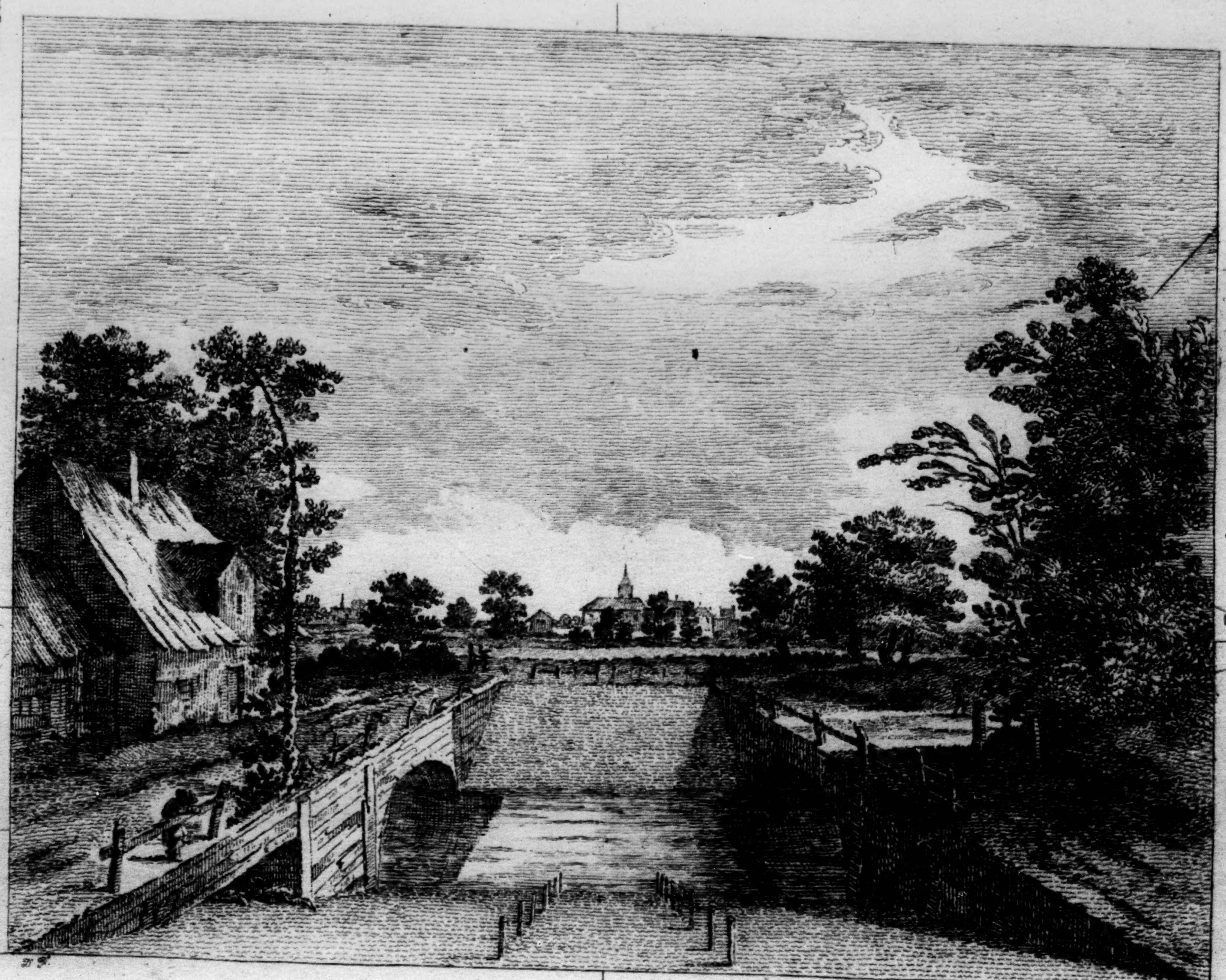
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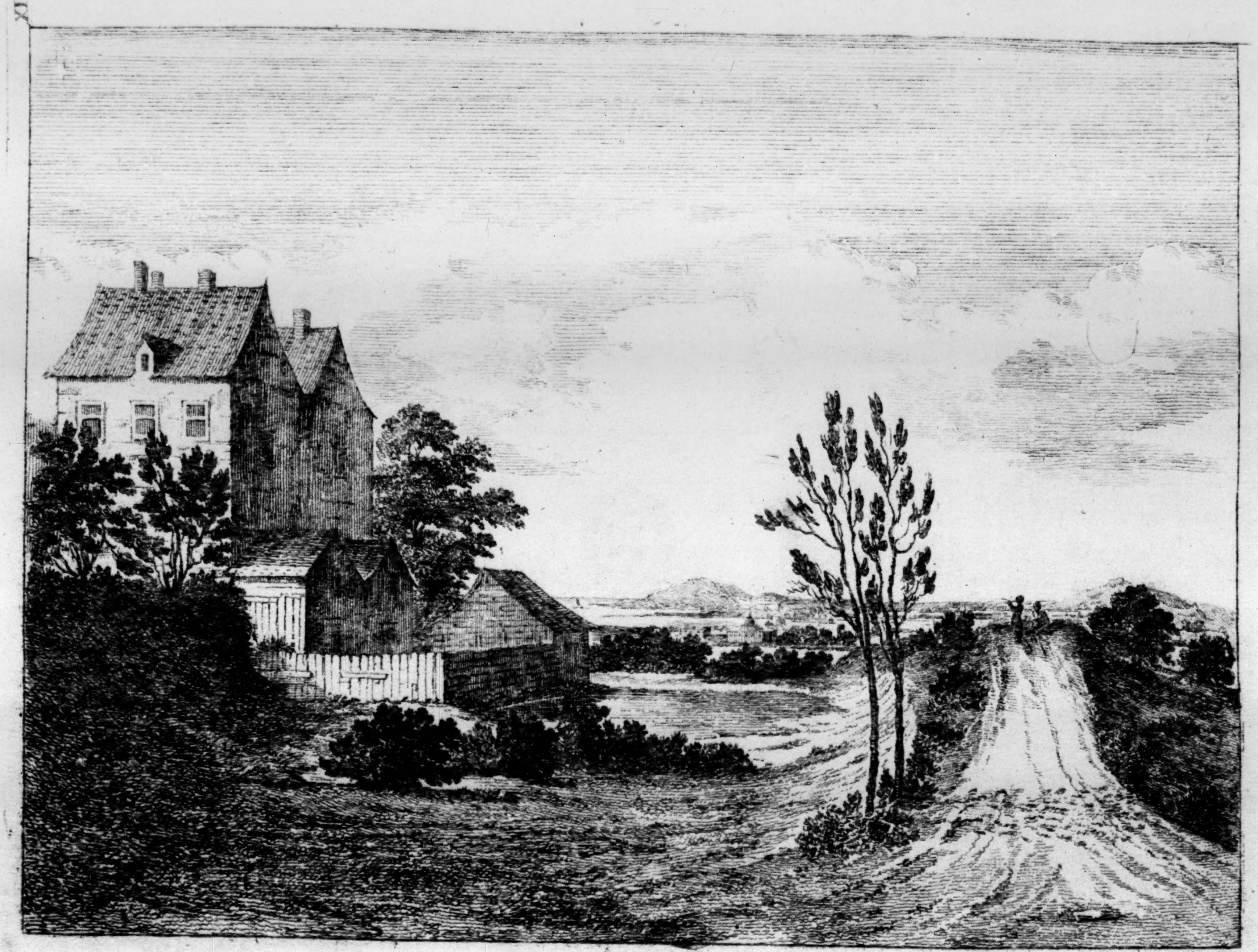

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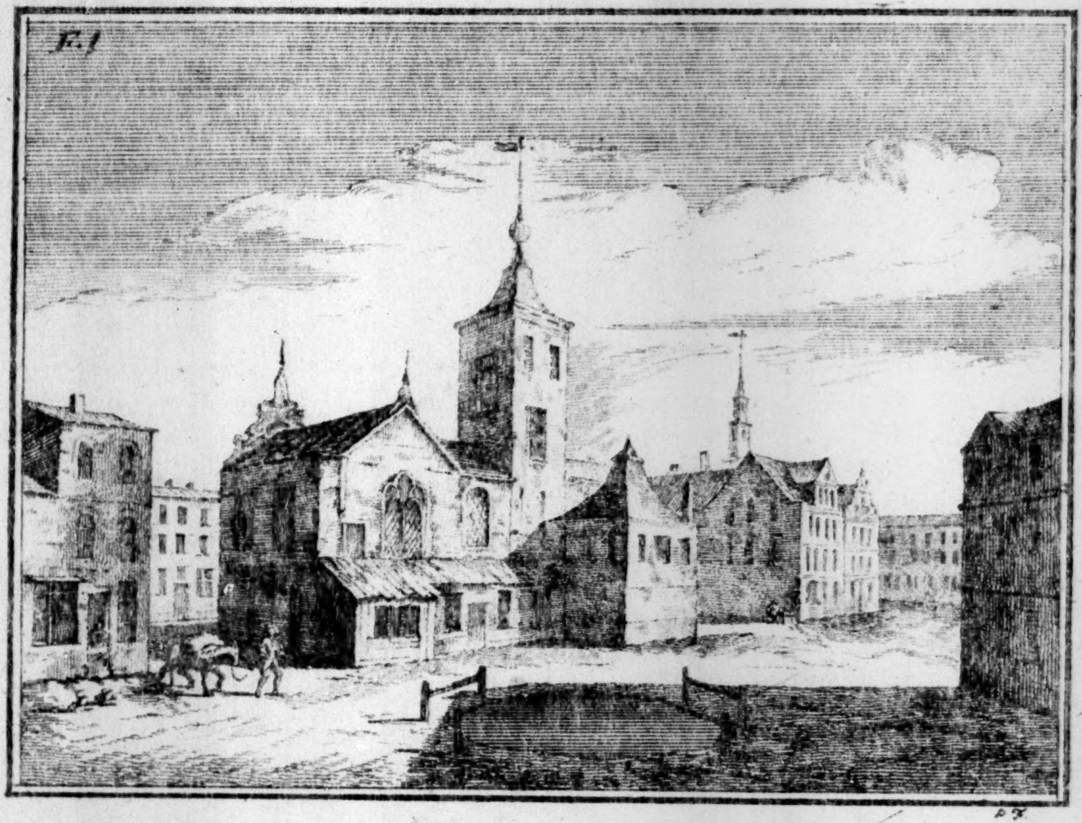

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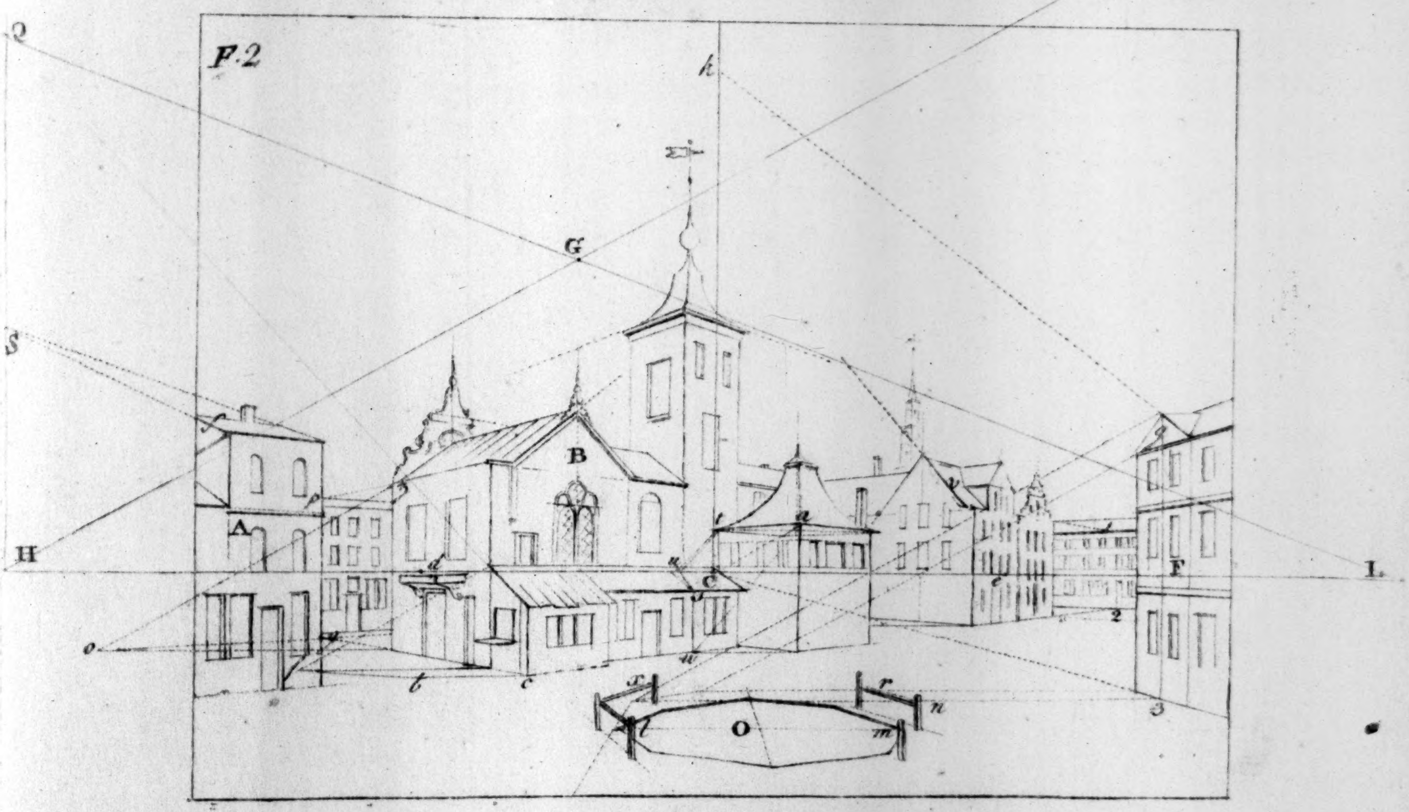

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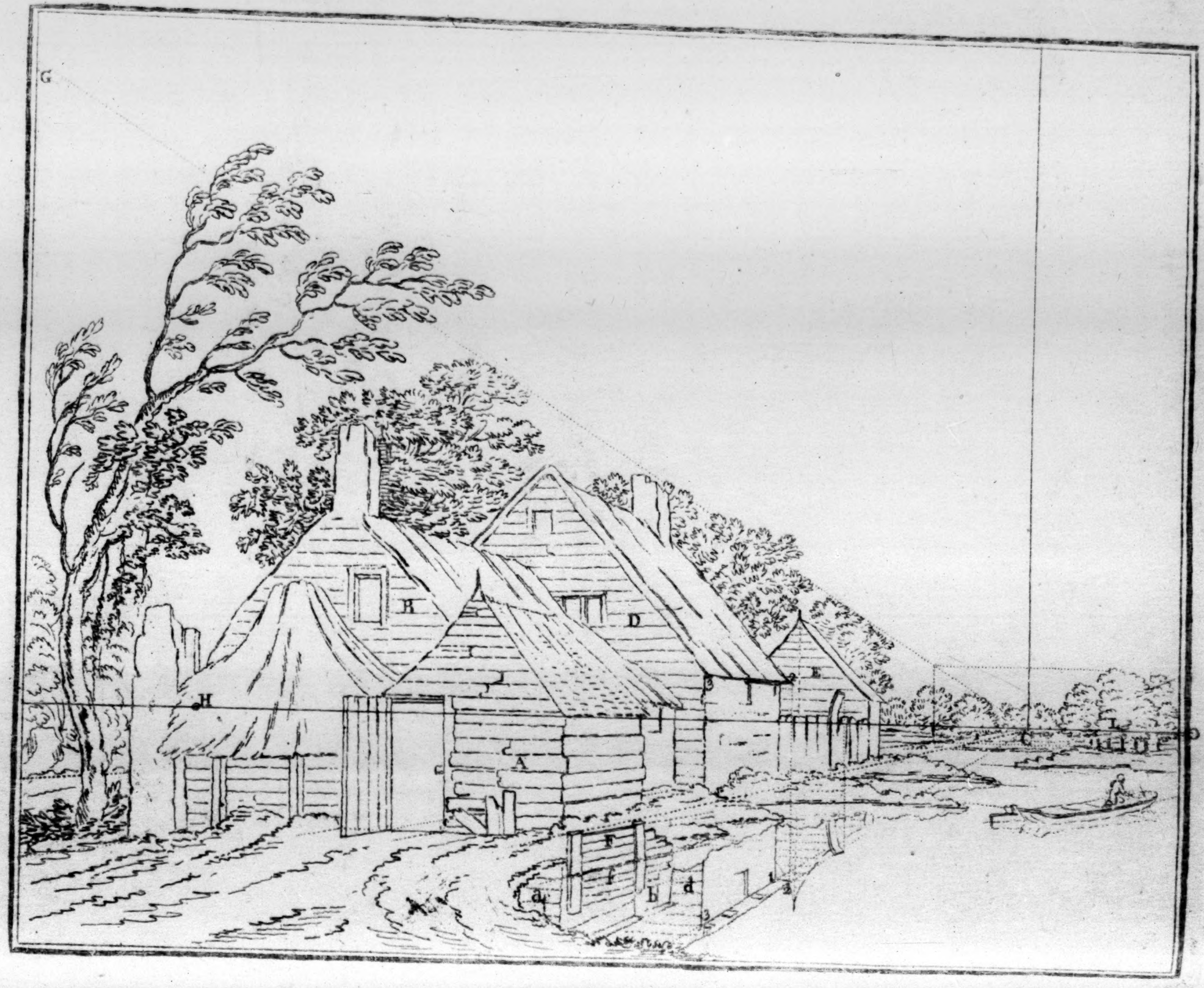

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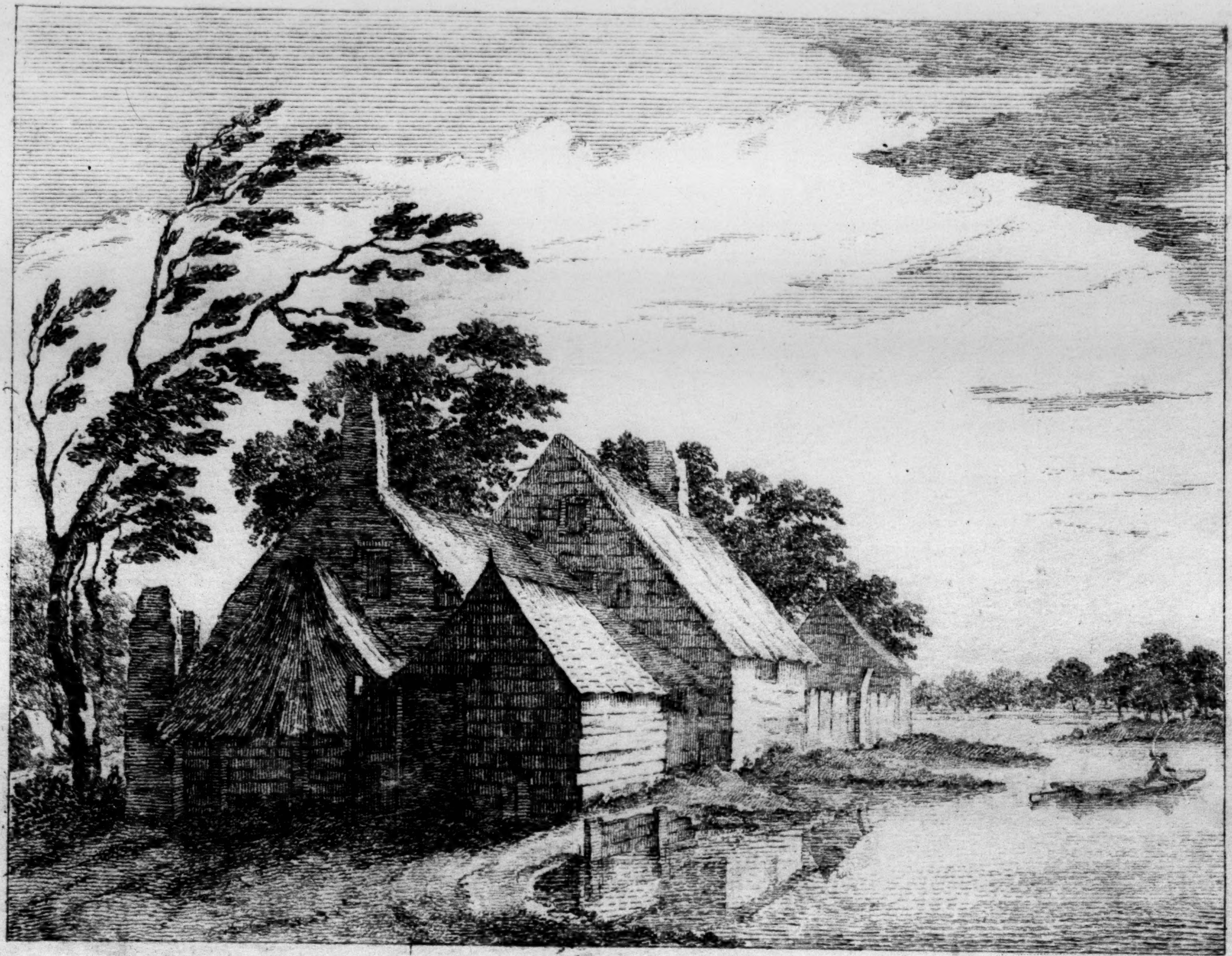

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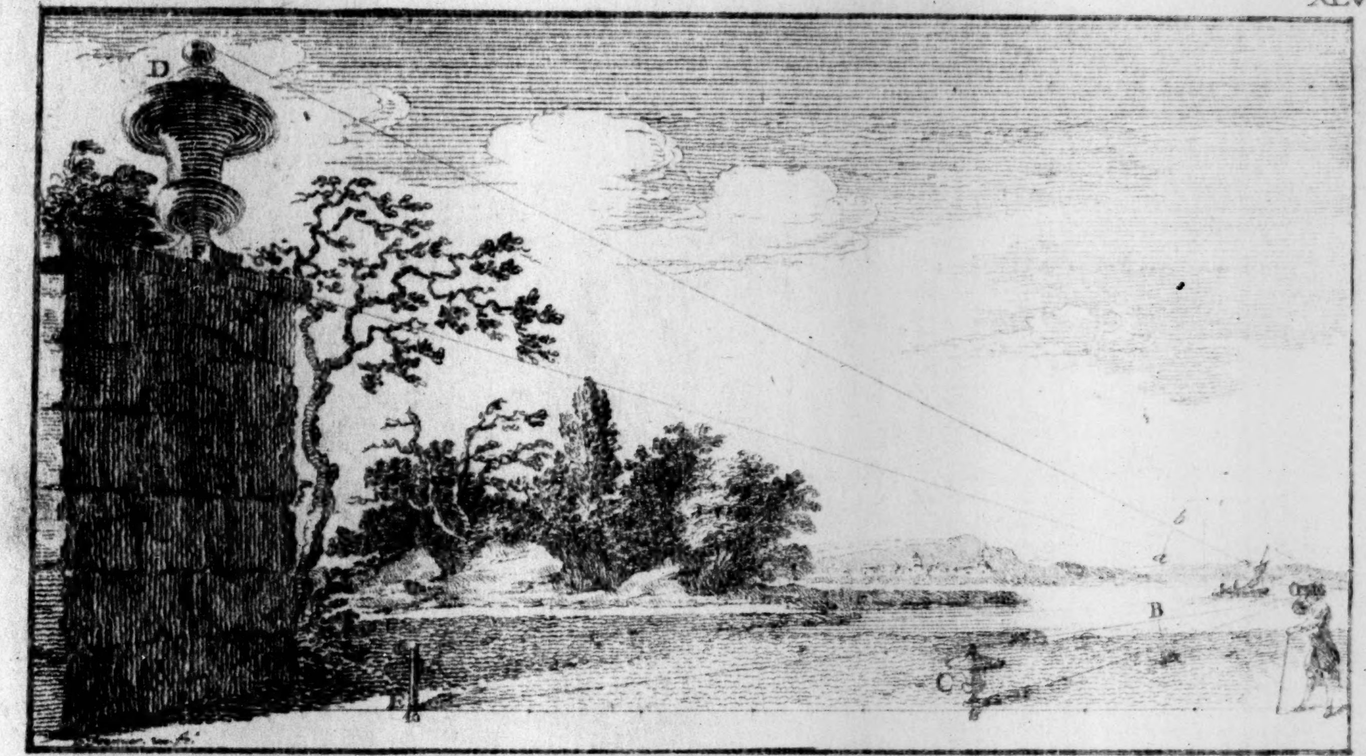
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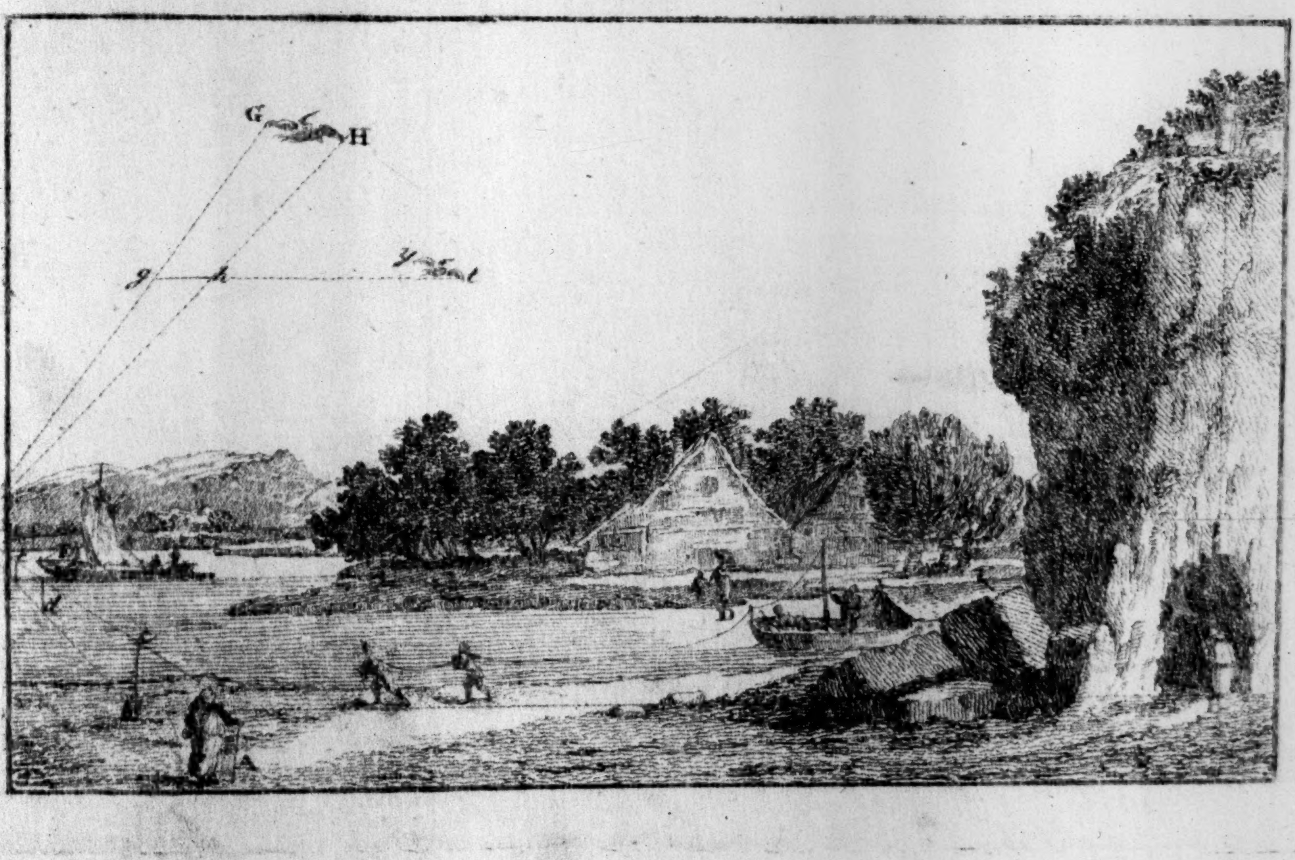
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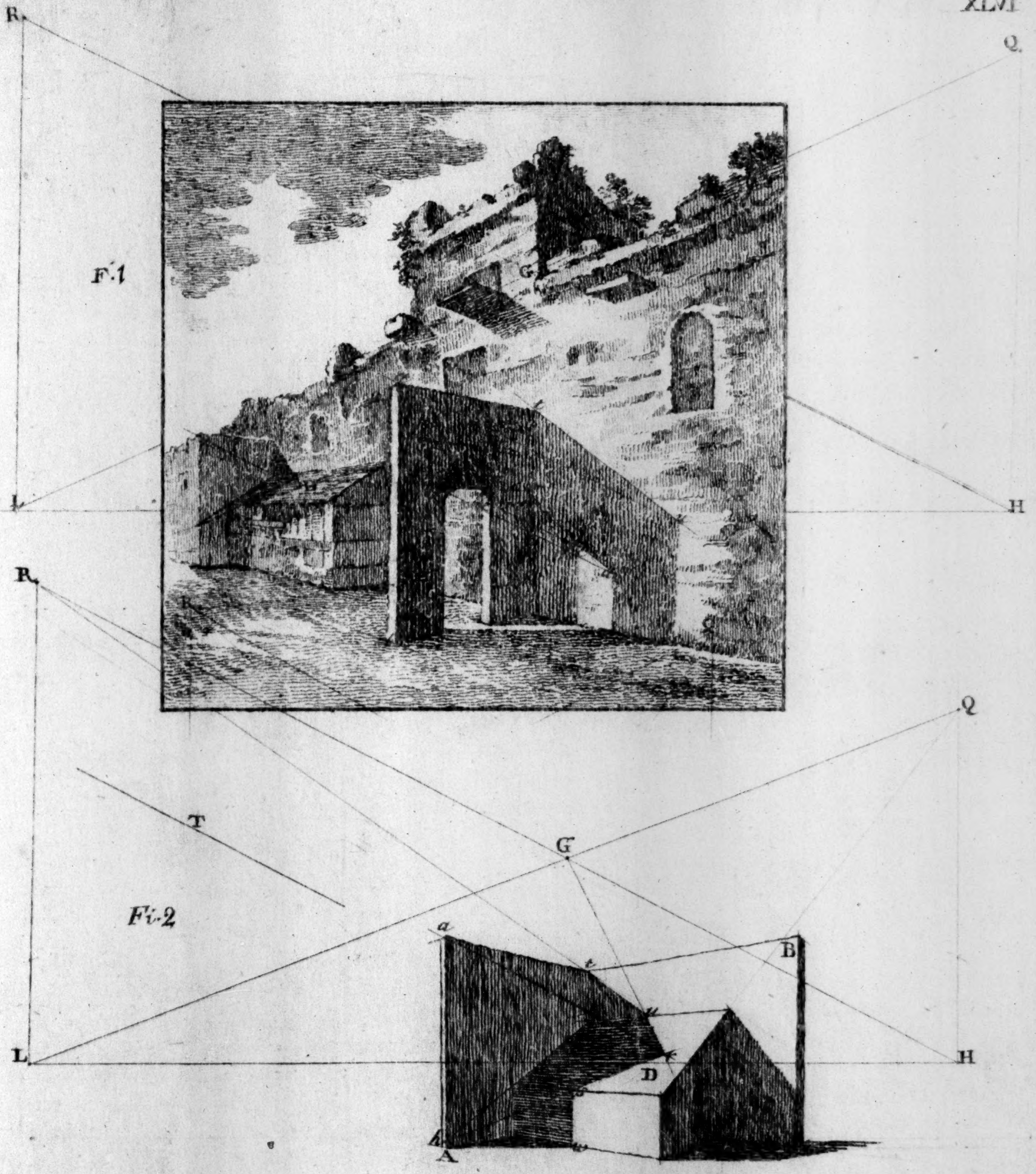
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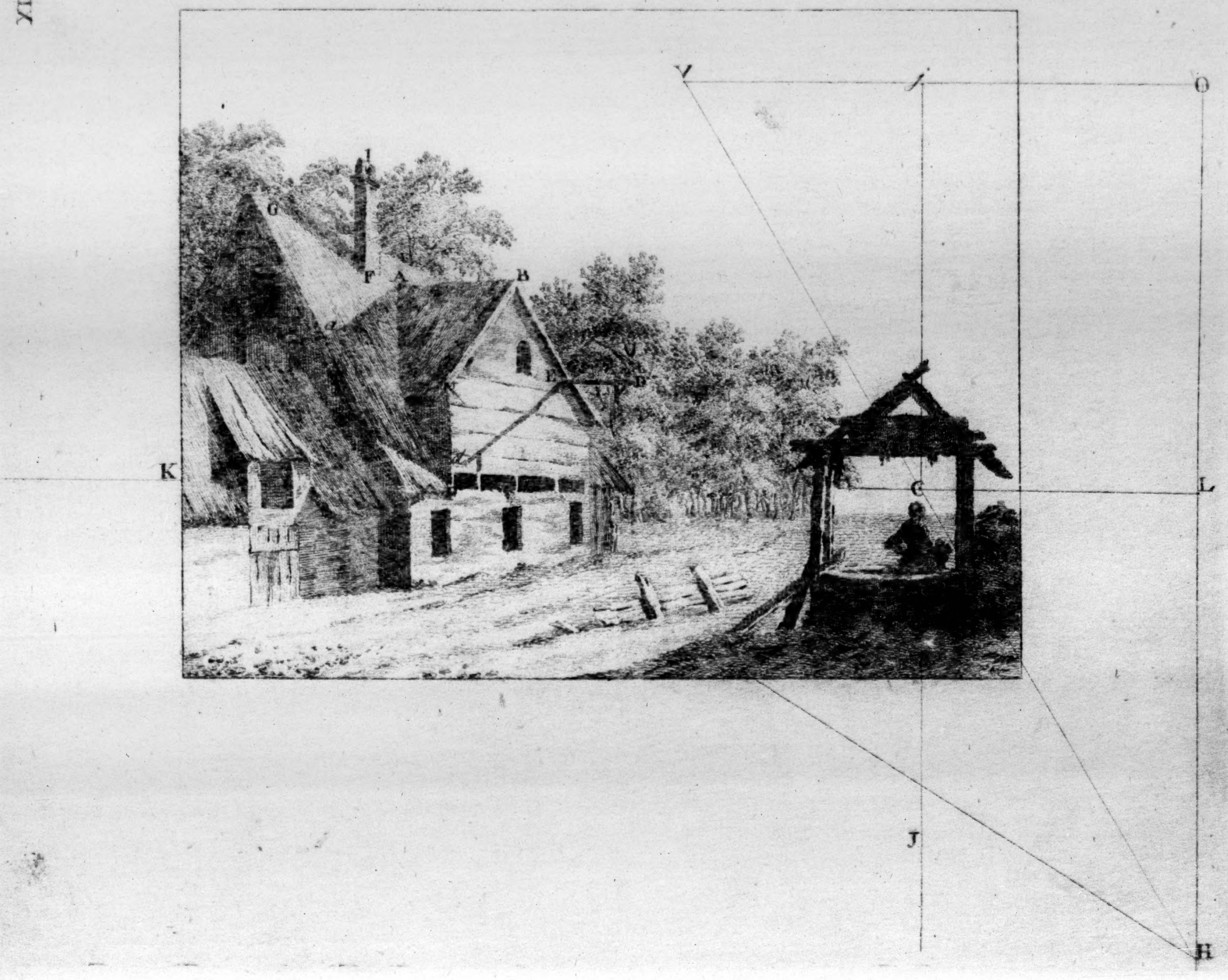
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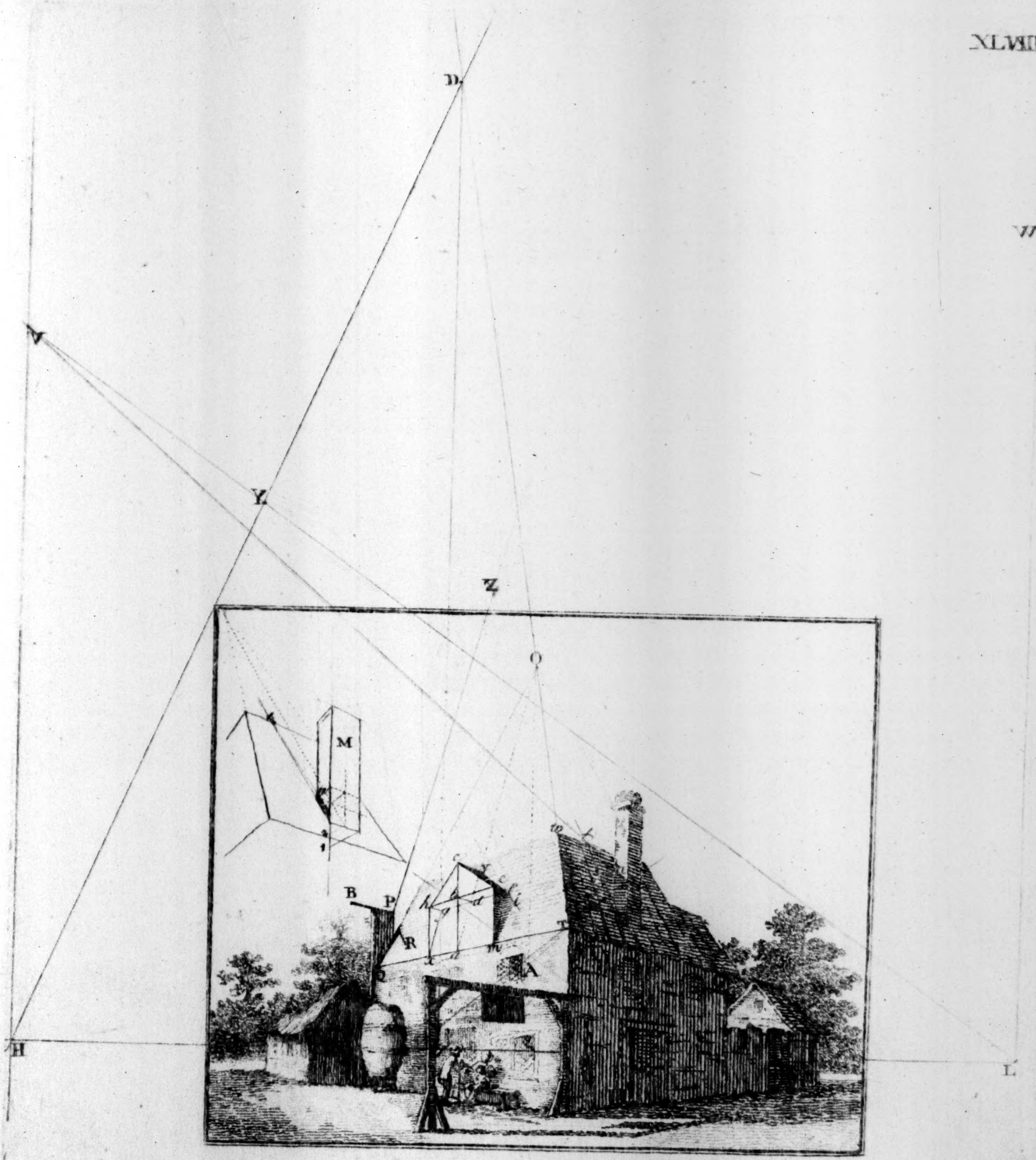
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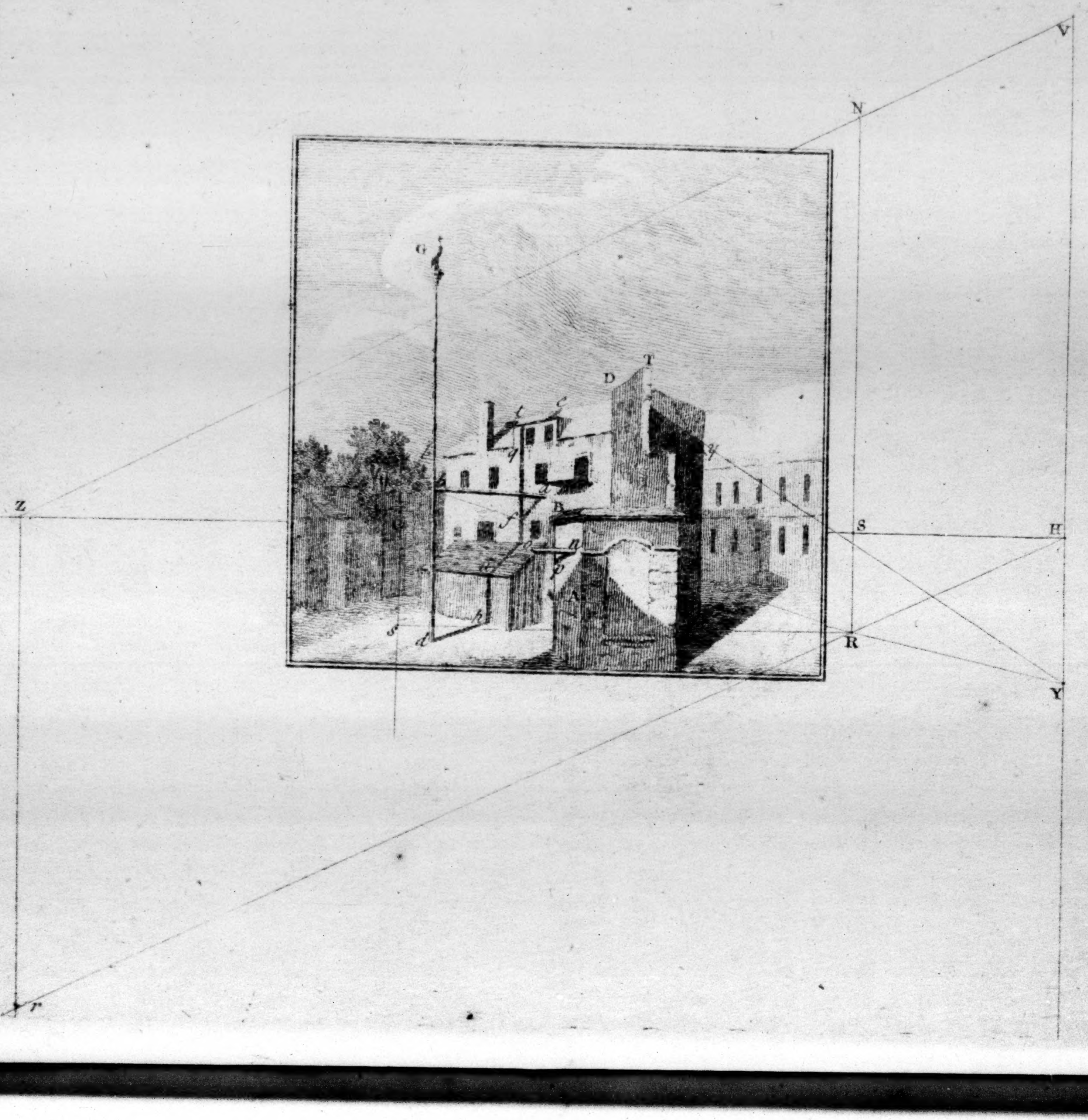
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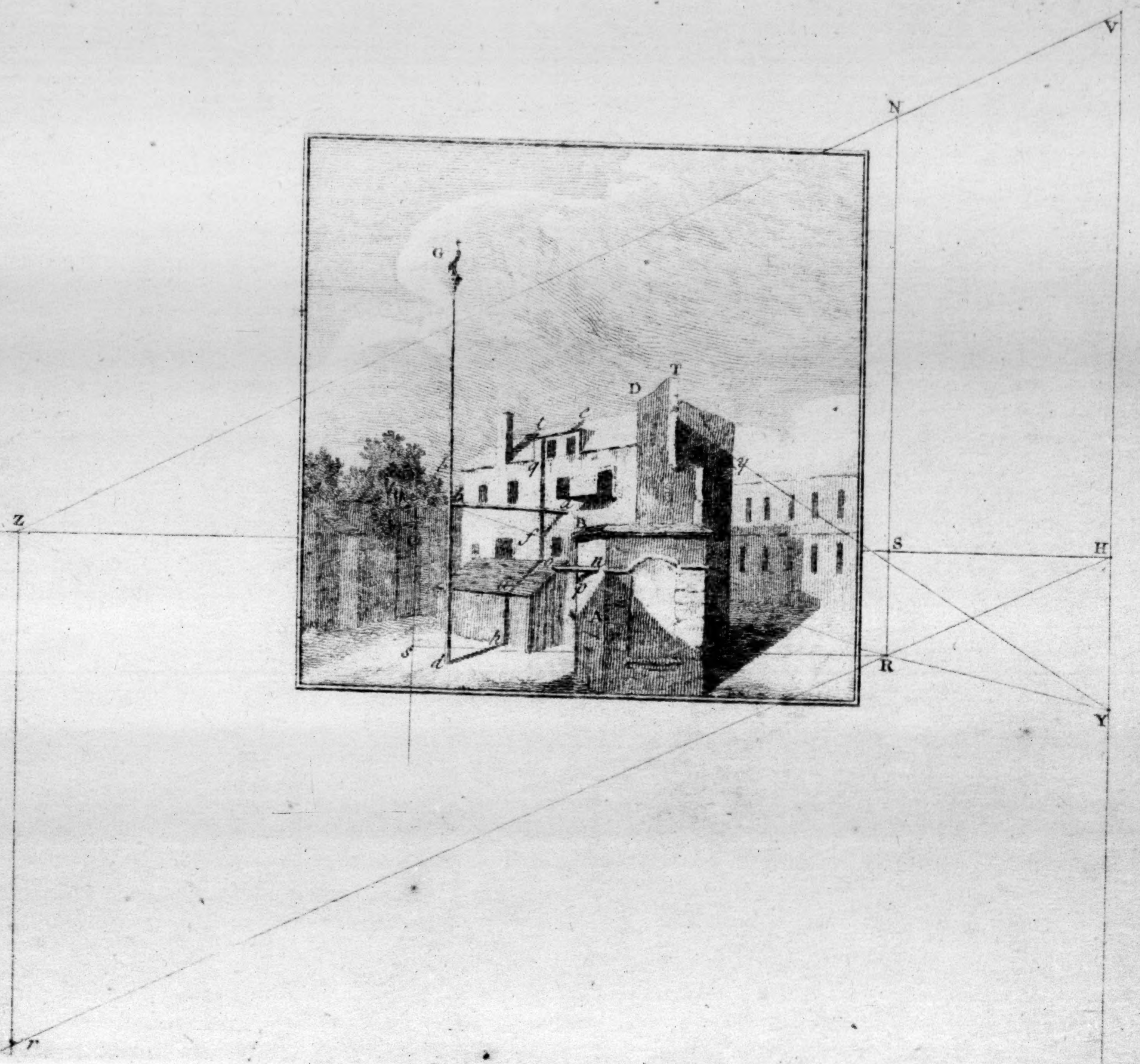

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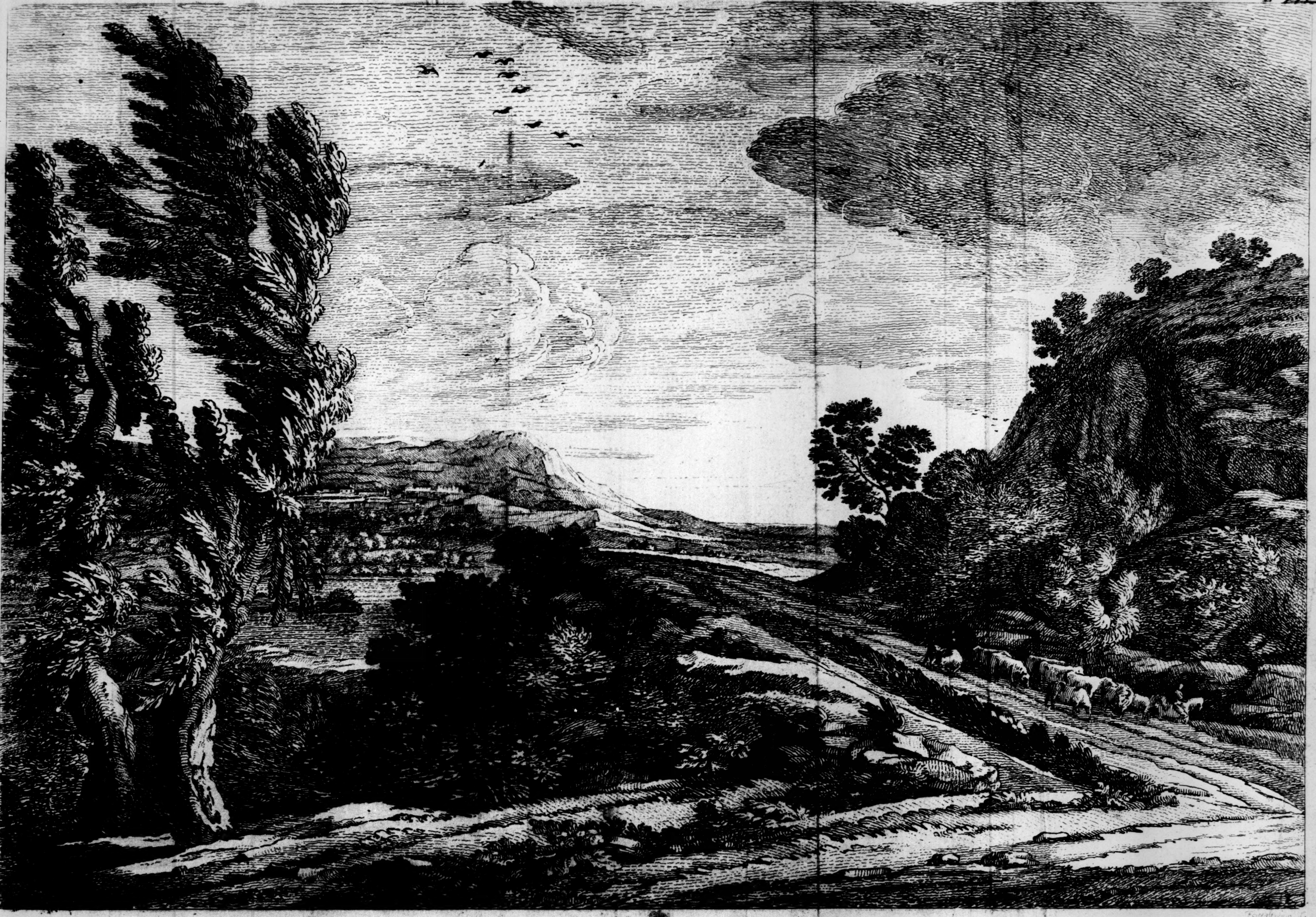

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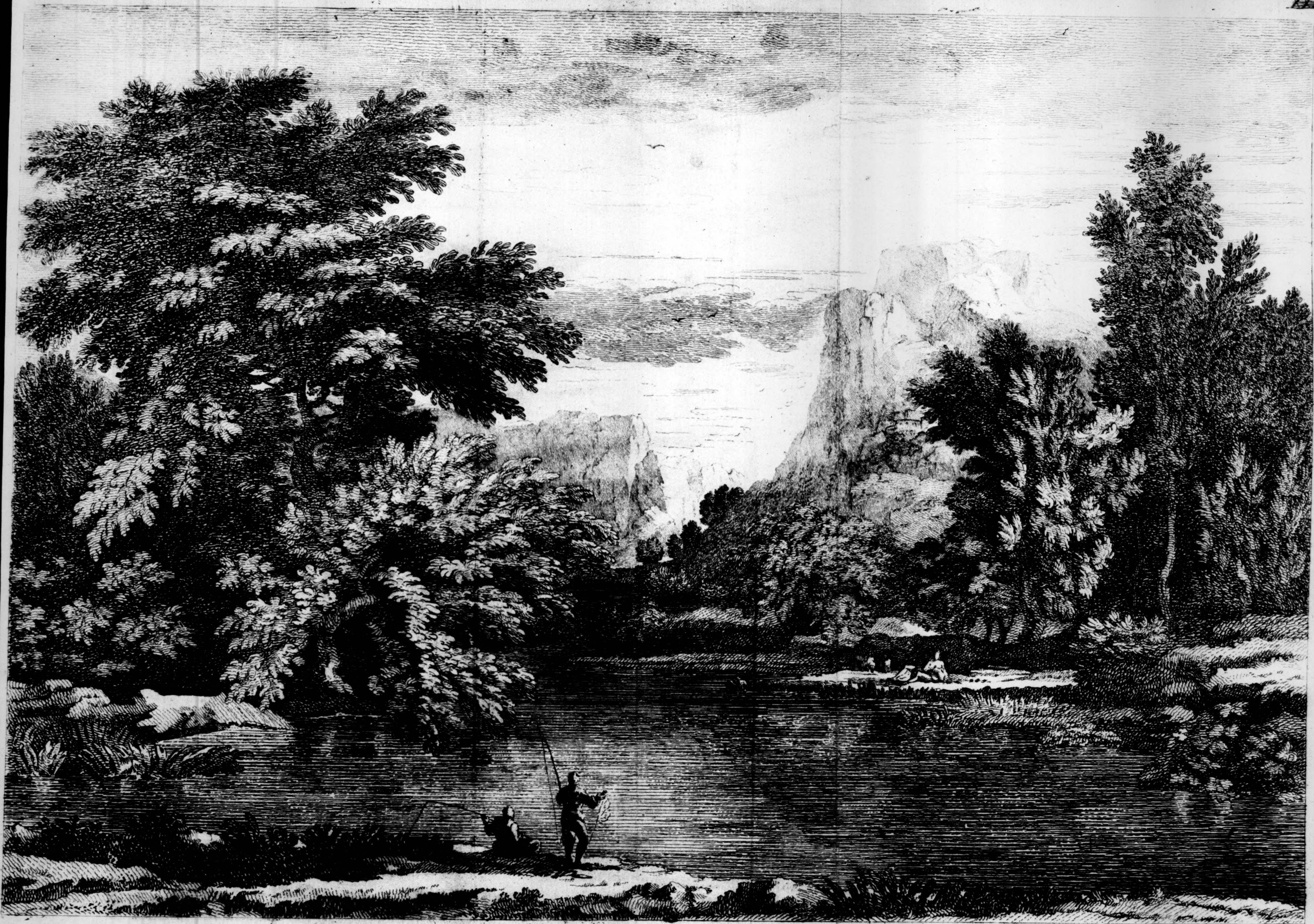

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